THE JOURNAL OF FINANCE • VOL. XL, NO. 5 • DECEMBER 1985

A Model for the Determination of "Fair" Premiums on Lease Cancellation Insurance Policies

JAMES S. SCHALLHEIM and JOHN J. McCONNELL*

ABSTRACT

Lease cancellation insurance protects the lessor against early termination of a cancellable operating lease. This paper presents a contingent claims model for determining the "fair" premium for this type of insurance policy. Comparative statics are considered, and some numerical examples are presented to illustrate the model. Among other things, the insurance premium is sensitive to the expected rate of economic depreciation of the leased asset and to the leased asset's systematic and nonsystematic risk.

CURRENTLY, THERE ARE IN use two general types of asset leasing insurance: (1) residual value insurance and (2) lease cancellation insurance. Residual value insurance has been in use for a relatively long period of time, and many insurers provide this type of coverage. The insurance is constructed so as to pay the insured—the lessor—the difference between a stated "insured amount" and the "residual" or market value of the leased asset at the maturity date of the lease. Lease cancellation insurance, on the other hand, is a relatively new product, and its availability is limited. The issuer of lease cancellation insurance essentially agrees to continue making rental payments under a cancellable lease if the lessee chooses to cancel the lease. In essence, lease cancellation insurance guarantees a flow of income to the lessor.

According to most accounts, lease cancellation insurance was originated by Lloyd's of London in 1974.[1] This insurance was initially written to cover lessors who leased computer equipment under cancellable operating leases. These leases grant the lessee the right to cancel the lease after a fixed number of rental payments. Essentially, the insurer offers protection to the lessor from early cancellation by guaranteeing the lease payments until the maturity date of the lease contract.

Lease cancellation insurance emerged from relative obscurity in late 1979 and early 1980. During that period, Lloyd's suffered the largest loss due to a single catastrophic event in its 270-year history. That event was the massive cancellation of leases for computer equipment which followed the introduction, by IBM,

* Graduate School of Business, University of Utah and Purdue University, respectively. In developing this paper, the authors have benefited from stimulating conversations with Kenneth Dunn, Robert Geske, and Steven Manaster. Comments by Sanjai Bhagat, Jim Brickley, Fikry Gahin, William Kracaw, Rene Stultz, and participants at finance workshops at the University of Utah and Washington State University also have been most helpful. Financial support for this research was provided to John McConnell by the Eli Lilly Corporation.

[1] See, e.g., Emmrich [10] and Cipolla and Spilka [7].

of a new generation of more efficient computer equipment. The actual lessors of these insured lease contracts were independent leasing companies. Two of the most prominent firms involved with the contracts were Itel Corporation and OPM Leasing Company who, despite the insurance coverage by Lloyd's, filed for bankruptcy shortly after the new IBM computers entered the market. Lloyd's potential losses were initially estimated to be as high as $600 million, but, after two years of litigation, Lloyd's eventually agreed to a settlement estimated to be approximately $250 million.[2]

IBM actually introduced its new generation of computer equipment in 1977, but, because the insured leases typically included three-year noncancellation periods, lease cancellations did not occur until two or three years later. However, in 1977, it became apparent that massive terminations were on the horizon, and in late 1977, Lloyd's discontinued writing lease cancellation insurance. In the wake of its disastrous experience with this type of insurance, Lloyd's was accused of "misestimating" the risk in lease cancellation insurance and of charging "too low" a premium for its policies.

Immediately following the decision by Lloyd's to discontinue issuing lease cancellation insurance, this type of insurance coverage disappeared from the market. However, after a hiatus of approximately three years, several U.S. insurers began to issue a policy very similar to the original Lloyd's policy. Currently, cancellation insurance is available for leases on various types of equipment, including computer equipment.

The purpose of this paper is to present a model for the determination of "fair" premiums on lease cancellation insurance. Once the model is developed, comparative statics are considered to demonstrate the qualitative effect of changes in the various characteristics of the lease contract, the leased asset, and the insurance contract on the amount of the insurance premium. Numerical examples are then presented to illustrate the quantitative effect of changes in the various characteristics of the least contract, the leased asset, and the insurance contract on the magnitude of the insurance premium.

The model presented here is a contingent claims valuation model and builds upon a model for evaluating asset leasing contracts developed by McConnell and Schallheim [20]. As such, it is one of several types of contingent claims models that could be developed to calculate competitive insurance premiums.[3] This model does, however, encompass most of the variables relevant to the determination of lease cancellation insurance premiums and, as a consequence, it could

[2] Various accounts of the suits, countersuits, and final settlement involving claims against Lloyd's for its lease cancellation insurance are contained in: "Lloyd's Biggest Disaster," *Forbes*, May 28, 1979, p. 28+; "Uncalculated Risk: At Lloyd's of London, A Record Loss Looms on Computer Policies," *Wall Street Journal*, July 10, 1979, p. 1+; "Computer Lease Losses Exceed Lloyd's Forecast," *The Journal of Commerce*, November 19, 1981, pp. 7–8; and "Bad Luck Forces Updating at Lloyd's of London," *Money and Banking*, February 25, 1980, pp. 94–108.

[3] Other recent papers that have employed asset pricing models to explore the characteristics of competitive insurance premiums include Brennan and Schwartz [4, 5], Gatto et al. [12], and Kraus and Ross [16].

be used by insurers for setting benchmarks in calculating lease cancellation insurance premiums.[4]

The remainder of the paper is organized as follows. Section I contains a description of the terms of a "typical" lease cancellation insurance policy currently in use. Section II gives a definition of a "fair" insurance premium and outlines a method for determining fair premiums under lease cancellation insurance. Section III summarizes and recapitulates the model developed in McConnell and Schallheim [20] for determining equilibrium rental payments on cancellable and noncancellable equipment leasing contracts. This model is the basis of the model for determining insurance premiums. Section IV considers the comparative statics of the competitive insurance premium and presents numerical examples that illustrate the effect of changes in various parameters of the model on the magnitude of the premium. Section V contains a discussion of the problems of moral hazard and adverse selection that may be latent in the structure of lease cancellation insurance policies. The final section is a conclusion.

I. Description of Lease Cancellation Insurance Contracts

Lease cancellation insurance is written to protect issuers of cancellable leases against disruption of the income stream provided by the lease. Cancellable leases, generally known as operating leases, call for fixed periodic rental payments and have fixed maturities, but they give the lessee the right to terminate the lease without penalty after a fixed number of payments have been made, but before the maturity date of the lease.

All lease cancellation insurance policies are not identical, but most contain the same basic provisions.[5] Under the standard lease cancellation policy, the full amount of the insurance premium is paid when the policy is issued, which typically coincides with the initiation of the lease. The insurer then agrees to compensate the lessor in the event the lease is terminated prematurely. When a lease is terminated, the lessor is responsible for reselling the asset in the secondary market or for leasing it to another lessee. In either case, the insurance company must approve the terms of the transaction before any payments are made to the lessor.

If the asset is relet, the insurer pays the lessor the difference between the periodic rental payment specified in the lease and the amount of the rental payment for which the asset is relet, less a deductible. In most policies, the deductible is expressed as a fraction of the rental payments due under the lease

[4] We leave unanswered the question of why lessors purchase lease cancellation insurance. Since most lessors are corporations, standard arguments for the purchase of personal insurance do not apply. It is possible that the arguments of Mayers and Smith [18, 19] concerning the corporate demand for other types of insurance could explain the use of lease cancellation insurance by corporations.

[5] We thank Seymor E. Spilka, President, Spilka Co., Forest Hills, NY, for providing us with copies of a number of lease cancellation insurance policies sold by several different insurers. We also benefited from several lengthy and informative telephone conversations with Mr. Spilka.

contract less the amount of the periodic rental payment for which the asset is relet. If the asset is resold, the insurer pays the lessor the "net loss" defined as the undiscounted sum of all remaining lease payments due under the lease less the amount for which the asset is resold less a deductible. In this case, the deductible is expressed as a fraction of the "net loss" incurred.

Lease cancellation policies frequently contain two further provisions that serve to limit the insurer's liability under the contract. First, a policy may contain a limit on the total payments to be made by the insurer to the lessor in the event of cancellation by the lessee. This provision limits the insurer's maximum liability. Second, in the event that the lease is cancelled and the asset is relet, the insurer receives a claim on the residual value of the leased asset equal to the total dollar amount of the payments made by the insurer to the lessor. In essence, this provision gives the insurer a claim to a rebate, at the maturity of the lease, to payments made under the insurance policy. This claim takes precedence over the lessor's (i.e., the asset owner's) claim to the residual value of the asset. We should emphasize that these last two provisions are variations on a theme, and they are not contained in all policies.

Finally, lease cancellation insurance does not indemnify the lessor against any reduction in the value of the leased equipment due to physical damage other than normal wear and tear. Indeed, in most instances, the lessee is required to carry separate casualty insurance to cover any physical damage to the leased asset. Lease cancellation insurance also does not indemnify the lessor against loss resulting from the financial insolvency of the lessee. For example, the policy does not insure the lessor in the event of bankruptcy by the lessee.

II. The Definition of "Fair" Insurance Premiums

The various provisions of lease cancellation policies specify the insurer's liability (or, alternatively, the payments to be received by the lessor) under various possible contingencies as spelled out in the insurance policy. An appreciation of the provisions of the insurance policy is useful because it is these provisions, along with the characteristics of the leased asset and the covenants of the lease, that determine the amount of the fair insurance premium, where, according to our definition, a fair premium is one that precludes profitable arbitrage in a perfect and competitive capital market.[6]

For the purposes of the analysis that follows, it is convenient to simplify the provisions of the lease cancellation insurance policy. The basic policy that we analyze:

1. calls for the premium to be paid in full when the policy is issued;
2. calls for the asset to be relet once cancellation occurs;
3. calls for full coverage of the lease payments by the insurer less a fractional deductible (i.e., there is no maximum limitation on the insurer's liability

[6] We defer until Section V a discussion of the problems of moral hazard and adverse selection that may exist in lease cancellation insurance policies.

other than the payments due under the lease and there is no rebate at maturity).

With this simplified version of the lease cancellation insurance policy, we are able to focus upon the primary determinants of the fair insurance premium in a competitive market.

Under the basic lease cancellation insurance policy, the insurer essentially agrees to continue making lease payments to the lessor in the event that the lessee cancels the lease. Thus, with lease cancellation insurance, the future stream of lease payments becomes a risk-free stream of income to the lessor. Given the amount of the periodic lease payments due under the lease and assuming that the risk-free rate of interest is known and constant for all future periods, the present value of the lease payments can be determined by discounting them at the risk-free rate of interest. That is, once the insurance premium is paid, the lease payments under the cancellable lease become riskless to the lessor, and the value of the stream of payments can be determined as

$$V(L^C) = \sum_{i=0}^{T-1} L^C \cdot R_f^{-i} \tag{1}$$

where L^C is the equilibrium rental payment under a cancellable lease which covers T periods, R_f is one plus the risk-free rate of interest, and i denotes time periods. Under the lease, the first rental payment is due at the beginning of period 1 (i.e., at $i = 0$), and the last payment is due at the beginning of period T (i.e., at $i = T - 1$).

Given that the lease payments under the cancellable lease are actually risky, the lease payments under a noncancellable lease, L^{NC}, would generally be less than those under a cancellable lease. As a consequence, the quantity $V(L^C)$ overcompensates the lessor for the risk borne because the lessor is no longer bearing the cancellation risk. The differential risk is borne by the insurer.

Given that the lease payments under a noncancellable lease are, in fact, risk-free, the present value of those payments can be determined by discounting them at the risk-free rate of interest.[7] Let this amount be

$$V(L^{NC}) = \sum_{i=0}^{T-1} L^{NC} \cdot R_f^{-i}. \tag{2}$$

Then, the amount of excess compensation to the lessor is $V(L^C) - V(L^{NC})$, and, because the insurer is now bearing the risk of lease cancellation, the insurance premium can be determined as

$$IP = V(L^C) - V(L^{NC}). \tag{3}$$

Thus, in this framework, the determination of the fair insurance premium is a three-step procedure. First, determine the equilibrium rental payment, L^C, appropriate for the lease that it is to be insured. Second, determine the equilibrium rental payment appropriate for a noncancellable lease for the same asset and of

[7] This assumes that noncancellable (i.e., financial) leases are, in fact, noncancellable and that the probability of bankruptcy by the lessee is either zero or is unaffected by the choice of a cancellable or noncancellable lease. For further discussion of this point, see McConnell and Schallheim [20].

the same maturity as that of the lease to be insured. And finally, discount the two sets of rental payments at the risk-free rate of interest. The difference between the two present values is the fair insurance premium.[8,9]

The insurance premium determined in this way is appropriate for a fully insured lease. The various deductibles and limitations that appear in policies reduce the value of the insurance and, as a consequence, reduce the amount of the fair premium. The simplest convenant to incorporate is the fractional deductible. This feature is simple to incorporate because the fractional deductible reduces the insurer's liability proportionately under each possible outcome. Let α be the amount of the fractional deductible. Then the insurance premium appropriate for a policy with a fractional deductible is

$$IP = (1 - \alpha)[V(L^{C}) - V(L^{NC})]. \tag{4}$$

With a deductible of say 10 percent, the insurer's coverage is only 90 percent of the coverage without the deductible, and the insurance premium is only 90 percent as well.

[8] The proof of this relationship can be established with a simple cashflow dominance argument. To see this, consider the cashflows to the lessor that issues a cancellable lease and purchases lease cancellation insurance. Let us suppose that the term to maturity of the lease is three periods. At time $i = 0$, the lessor purchases an asset for the amount A_0, pays the insurance premium IP, and receives the first lease payment, L^C. Lease payments in the amount of L^C are also received at times $i = 1$ and $i = 2$. At time $i = 3$, the lessor receives the uncertain residual value of the asset, $\tilde{S}V$, where $\sim$ indicates a random variable. Now, consider the cashflows to the lessor that issues a noncancellable, three-period lease. At time $i = 0$, the lessor purchases the asset for A_0 and receives the first lease payment, L^{NC}. Lease payments of L^{NC} are also received at times $i = 1$ and $i = 2$. At time $i = 3$, the lessor receives the uncertain residual value of the asset, $\tilde{S}V$. The cashflows can be portrayed as:

	$i = 0$	$i = 1$	$i = 2$	$i = 3$
Cashflow to lessor with insured cancellable lease	$-A_0 - IP + L^C$	$+L^C$	$+L^C$	$+SV$
Cashflow to lessor with noncancellable lease	$-A_0 + L^{NC}$	$+L^{NC}$	$+L^{NC}$	$+SV$
Differential cashflow	$-IP + L^C - L^{NC}$	$+L^C - L^{NC}$	$+L^C - L^{NC}$	0

In order for all three contracts—the noncancellable lease, the cancellable lease, and cancellation insurance—to exist in equilibrium it must be that the net value of the differential cashflows between the two alternatives is zero. That is, it must be that $V(IP) - V(L^C - L^{NC}) = 0$ where V is a general valuation operator. Since the stream of rental payments, L^{NC}, is risk-free due to the noncancellability of the lease and the stream of rental payments, L^C, becomes risk-free once the insurance premium is paid, then

$$IP = \textstyle\sum_{i=0}^{T-1} L^C \cdot R_f^{-i} - \sum_{i=0}^{T-1} L^{NC} \cdot R_f^{-i}.$$

If this relationship does not hold, one of the contracts will be dominated and will not exist in equilibrium.

[9] An alternative, but identical, conceptualization of the insurance premium is as an American put option with discrete exercise dates. We have chosen not to model the cancellation insurance as an American put because doing so requires that the exercise price, L^C, be exogeneously determined. Our model, on the other hand, determines the equilibrium payment, L^C, endogeneously. This difference becomes important when we examine the effect of the changes in the various relevant parameters on the size of the insurance premium. This put characterization of the insurance premium does, however, point out the similarity between residual value insurance policies, which are simple European put options, and the lease cancellation insurance policies modeled here. (For further discussion of put valuation, see Geske and Johnson [14]).

The other three provisions that could be incorporated into the determination of the premium would be the resale option, the total renewal liability, and the rebate at the maturity of the lease of prior payments made by the insurer if the lease is cancelled. Rather than doing so, we will merely note that, as with the fractional deductible, each of these covenants serves to reduce the insurer's liability and, therefore, the insurance premium. In this regard, then, the *IP* of Equations (3) and (4) represent the upper bound on the insurance premium without and with a fractional deductible.

III. A Model for the Determination of Fair Insurance Premiums

Regardless of whether the policy contains a deductible, calculation of the insurance premium for a lease cancellation insurance policy continues to be the three-step procedure outlined above. The only difference is that the premium is reduced proportionately when the policy contains a fractional deductible. Thus, the key to determining the insurance premium is the determination of competitive rental rates for cancellable and noncancellable leases.

In a previous paper, McConnell and Schallheim [20], we developed a model for determining competitive rental payments for cancellable and noncancellable leases. That model considers the cancellable portion of the lease to be a compound call option on the use of the leased asset. On the date that each rental payment is due, the lessor may choose to exercise the option to retain the use of the leased asset by paying the fixed rental payment, where the fixed rental payment is the exercise price of the option. For leases that contain more than one cancellation opportunity, the payment of each lease payment purchases the use of the leased asset plus another option. Thus, the cancellable lease can be viewed as a compound option in the same spirit in which Geske [13] characterizes a risky coupon bond.

The model developed for the valuation of cancellable leases begins with the Miller and Upton [21] analysis of single-period leases and uses the valuation techniques of Rubinstein [23] and Geske [13] to expand the analysis to value multiperiod leases. Following these authors, the standard assumptions utilized are that investors are nonsatiated and risk-averse, that markets are perfect and competitive, and that no arbitrage opportunities exist. In addition, all asset returns are assumed to be distributed jointly lognormal with aggregate wealth; all investors are assumed to exhibit constant proportional risk aversion, so that investor demands can be aggregated. Furthermore, for convenience, the distribution of the rate of economic depreciation of the leased asset is assumed to be stationary over time, and the risk-free rate of interest is assumed to be constant.

The valuation model that follows from these assumptions can be used to value a variety of types of leasing contracts. The particular type of lease that is of interest in this paper is a T-period lease that calls for T lease payments of L^C each. The first lease payment is due on the origination date of the lease ($T = 0$), and the last is due at the beginning of period $T - 1$. The lease is noncancellable for the first K periods. Thus, the lessee is required to make the first K payments. Beginning with lease payment $K + 1$, the lessee may cancel the lease at any time.

The equilibrium rental for this type of lease can be expressed as

$$\begin{aligned} L^C = {} & (1 - \lambda^{T-K})A_0 - L^C \textstyle\sum_{i=1}^{T-K-1} R_f^{-i} \\ & + \textstyle\sum_{i=T-K}^{T-1} \lambda^i(1 - \lambda)A_0 \cdot N_{i-T+K}\,(h_{i-T+K} + \sigma\sqrt{i - T + K};\, \{\rho\}) \\ & - L^C \textstyle\sum_{i=T-K}^{T-1} R_f^{-i} \cdot N_{i-T+K}(h_{i-T+K};\, \{\rho\}) \end{aligned} \tag{5}$$

where L^C is the equilibrium lease payment, A_0 is the initial market value of the leased asset, R_f is one plus the risk-free rate of interest, $N_i(\cdot)$ represents the i-variate cumulative normal distribution with limits of integration h_i and correlation matrix $\{\rho\}$, and $\lambda = [(1 - \bar{d})/(1 + r_f)]e^{\sigma_{ly}}$ where $\bar{d}$ is the expected rate of economic depreciation of the leased asset, r_f is the risk-free rate of interest, and σ_{ly} is the covariance between the logarithm of one minus the rate of economic depreciation and the "market factor," y.[10] (For a formal derivation of Equation (5), see McConnell and Schallheim [20].)

Interpretation of the terms in Equation (5) is useful because it is those terms that are relevant to the determination of the insurance premium. The first two terms on the right-hand side of Equation (5) represent the value of the equilibrium lease payment during the noncancellation period of the lease. The equilibrium rental during the noncancellation period is a function of the initial value of the asset, A_0, the expected depreciation rate of the asset, $\bar{d}$, the risk-free rate of interest, r_f, and the covariance between the logarithm of one minus the expected rate of economic depreciation, $(1 - \bar{d})$, and the "market factor", y. Thus, the only "risk" that is relevant to the determination of the rental payment during the noncancellation period is the covariance risk or nondiversifiable risk associated with the change in the market value of the asset through time.

The third and fourth terms on the right-hand side of Equation (5) represent the value of the lease payments during the period in which the lease is cancellable. Over this period, the amount of the lease payment depends upon the parameters described above, but it additionally depends upon the probability of cancellation at each of the T possible cancellation points. These probabilities are subsumed in the i-variate cumulative normal probability distribution function, N_i. A critical variable in the determination of N_i is the variance rate of change in the asset's market value through time, σ^2 (see footnote 10).

Thus, given estimates of the appropriate parameters, Equation (5) can be used to perform the first step in the determination of the insurance premium for a lease which contains a cancellation option. The second step requires the determination of the equilibrium rental for the same asset under a fully noncancellable lease. However, as shown in McConnell and Schallheim [20], this type of lease

[10] For mathematical definitions of these terms, see Geske [13] or McConnell and Schallheim [20]. Note that h_i is analogous to the familiar cumulative normal upper bound of option pricing models, i.e., $h_i = (\ln(\lambda^i A_0/\bar{A}_i) + (\ln R_f - \sigma^2/2)i)/\sigma\sqrt{i}$ where $\bar{A}_i$ determines the boundary condition at each point in time i for cancellation/noncancellation in terms of the stochastic asset price.

The lease payment, L^C, is endogenous to the model. The first lease payment (which is, of course, L^C) contains the value of each future call option. However, the value of the future call options are dependent upon the exercise price, which is also L^C. Thus, L^C appears on both sides of Equation (5), and the solution for L^C requires an iterative computation technique.

is a special case of a cancellable lease. When the lease is noncancellable for the entire life of the contract, Equation (5) reduces to

$$L^{NC} = (1 - \lambda^T)A_0 - L^{NC} \sum_{i=1}^{T-1} R_f^{-i}, \tag{6}$$

and the data that are required to calculate the amount of the lease payment under the cancellable lease are also appropriate for calculating the amount of the lease payment under the noncancellable lease.

Given the amount of the cancellable lease payment, L^C, from Equation (5) and the amount of the noncancellable lease payment, L^{NC}, from Equation (6) and the risk-free rate of interest, r_f, the insurance premium on a lease cancellation policy with fractional deductible, α, can be computed as

$$\begin{aligned} IP &= (1 - \alpha)[V(L^C) - V(L^{NC})] \\ &= (1 - \alpha) \sum_{i=0}^{T-1} L^C \cdot R_f^{-i} - \sum_{i=0}^{T-1} L^{NC} \cdot R_f^{-i} \\ &= (1 - \alpha) \sum_{i=0}^{T-1} (L^C - L^{NC}) \cdot R_f^{-i} \\ &= (1 - \alpha)(L^C - L^{NC}) \sum_{i=0}^{T-1} R_f^{-i}. \end{aligned} \tag{7}$$

IV. Comparative Statics and Numerical Analysis of the Fair Insurance Premium

From Equation (7), the fair insurance premium will depend upon the characteristics of the capital market equilibrium, the characteristic of the leased asset, the terms of the lease, and the terms of the insurance policy. The qualitative effect of the various terms can be illustrated by considering the partial derivatives of the premium with respect to the various relevant parameters. The quantitative effect of the various terms can be illustrated by means of some numerical examples. Additionally, because certain of the partial derivatives are of indeterminate sign, the numerical examples are useful for indicating the sign of the changes in the insurance premium in response to changes in the parameter of interest over various ranges of the parameter.

A. *Partial Derivatives*

Interpretation of the signs of the partial derivatives is not always straightforward because the insurance premium is the difference between the present values of two streams of cash flows, each of which depends upon the same parameters.[11] Thus, the effect of a change in any parameter on the insurance premium depends upon the net effect of the change in the parameter upon the two cash flow streams, L^C and L^{NC}. In addition, the effect of any one parameter on the insurance premium may very well depend upon the value of the other parameters. With

[11] The cancellable lease payment, L^C, depends directly upon the variance rate of change in the asset's value and upon the covariance between the asset value and the market factor. The noncancellable lease payment, L^C, depends upon the covariance term, but does not depend upon the variance term once the covariance term is taken into account.

this in mind, it is possible to give some interpretation to the response of the insurance premium to changes in the values of its arguments.

(1) As the asset price increases so does the insurance premium; $\partial IP/\partial A_0 > 0$. Both L^C and L^{NC} are proportional to the initial market value of the leased asset so that changes in A_0 induce proportional changes in L^C and L^{NC}, which, in turn, imply a proportional change in IP. This means, of course, that insurance premiums can be quoted as a dollar amount per dollar of the cost of the leased asset.

(2) As the variance rate increases so does the insurance premium; $\partial IP/\partial\sigma^2 > 0$. Calculation of L^C is based on a compound call option valuation model. As with most options, an increase in the volatility of the underlying asset's market value increases the value of the option. The increase in the value of the option is reflected in an increase in L^C which, in turn, increases the insurance premium.[12]

(3) As the expected rate of economic depreciation increases, so does the insurance premium; $\partial IP/\partial\bar{d} > 0$. An increase in the expected rate of economic depreciation increases both L^C and L^{NC}. However, L^{NC} increases only because the expected residual value of the leased asset at the maturity date of the contract declines with an increase in $\bar{d}$, whereas L^C increases due to the lower expected residual value of the leased asset at the maturity date of the contract *and* to an increased probability of cancellation at each rental payment date. The differential rate of change in L^C and L^{NC} leads to an increase in the difference between the two, which leads to an increase in IP.

(4) As the number of noncancellable lease payments increases, the insurance premium declines; $\partial IP/\partial K < 0$. An increase in the number of noncancellable rental payments (holding constant the *total* number of rental payments due under the lease) reduces the insurer's potential liability and, thereby, leads to a reduction in the insurance premium. In the limit, $K = N$ and the insurer bears no risk.

(5) As the covariance term increases, the insurance premium declines; $\partial IP/\partial\sigma_{ly} < 0$. As in standard capital market theory, the covariance term reflects the nondiversifiable risk associated with the leased asset. Because the covariance is defined in terms of the rate of economic depreciation of the leased asset rather than in terms of the more standard rate of capital appreciation, a change in the covariance term gives rise to a change of the opposite sign in L^C and L^{NC}. Furthermore, the net impact of a change in σ_{ly} on L^C and L^{NC} is such that the net effect will be a change in the insurance premium with a sign opposite the sign of the change in the covariance term.

(6) As the deductible increases, the insurance premium declines; $\partial IP/\partial\alpha < 0$. An increase in the proportional deductible leads to a decrease in the insurer's liability.

(7) As the term-to-maturity of the lease contract increases, the insurance premium may increase or decrease; $\partial IP/\partial T \lesseqgtr 0$. As the maturity of the lease

[12] As discussed in footnote 11, it is difficult to separate the effect of the variance and covariance terms. However, if the insurance premium is characterized as an American put option (see footnote 9), then it is straightforward to see that the value of the insurance premium increases as the variance rate increases.

contract is extended, the rental payments become smaller both for the noncancellable leases and the cancellable leases, provided the number of cancellation options remains constant. However, the rates at which the two payments decline depend upon the values of the other relevant parameters. Thus, in some instances, the difference between L^C and L^{NC} becomes smaller as the number of lease payments is increased (resulting in a smaller insurance premium), and in some instances, the spread becomes larger as the number of payments is increased (resulting in a larger premium).[13]

(8) As the risk-free rate of interest increases, the insurance premium may either increase or decrease; $\partial IP/\partial r_f \lesseqgtr 0$. The indeterminate sign of the change in the insurance premium with respect to changes in the risk-free rate of interest is due to the confounding effect of changes in the risk-free rate on the size of the rental payments and on their present values. In the case of the cancellable lease payment, L^C, an increase in the risk-free rate will increase the lease payment. Therefore, on the one hand, an increase in the risk-free rate generally increases the spread between the cash flows L^C and L^{NC} at each date. On the other hand, an increase in r_f reduces the discounted value of the difference between the cash flows L^C and L^{NC}. The interaction between these two offsetting effects means that the insurance premium may go in either direction in response to an increase in the risk-free rate of interest.

B. Numerical Examples

In many cases, the sign of the partial derivatives of the insurance premium with respect to changes in the characteristics of the leased asset and to the terms of the lease contract are obvious, so that a formal model is not necessary to describe them. The same is not true with respect to the dollar amount of the insurance premium. Here, it is necessary to examine some numerical examples to gain insights into the relative effects of changes in the various parameters on the insurance premium.

In the illustrations that follow, the initial value of the leased asset is assumed to be \$1,000. Thus, the calculated dollar amount of the insurance premium could be converted to a fraction by dividing the premium by \$1,000, and this fractional amount could be used to compute the appropriate insurance premium for any "size" asset for which the values of the other relevant parameters are the same as those in the example. For convenience, the fractional deductible is assumed to be zero, but the premium in the examples could be adjusted for any deductible level merely by multiplying the premium by one minus the appropriate deductible.

Furthermore, the risk-free rate of interest is initially assumed to be 10 percent per year and the covariance term is assumed to be zero.

Table I focuses on the effect of changes in the variance rate of change in the leased asset's market value, σ^2, changes in the expected rate of economic depreciation, $\bar{d}$, and changes in the number of cancellation options under the lease,

[13] The discussion of the partial derivative with respect to the term-to-maturity parameter (T) assumes that the number of cancellable lease payments remains constant. If the number of cancellable lease payments were also to increase with T, then the insurance premium will surely increase due to the increased liability.

Table I

Insurance Premiums for Cancellation Insurance Policy: Sensitivity with Respect to Asset Variance Rate, Asset Rate of Economic Depreciation, and Number of Cancellation Options (Parameter Values: Initial Asset Cost = \$1,000; Risk-free Rate of Interest = 10% per year; and Covariance Rate = 0)

Variance Rate of Change in Market Value of Leased Asset (%/yr)	Insurance Premiums			
	Five-Year Lease		Seven-Year Lease	
	Two Cancellation Opportunities ($K = 3$)	Three Cancellation Opportunities ($K = 2$)	Two Cancellation Opportunities ($K = 5$)	Three Cancellation Opportunities ($K = 4$)
A. Expected Rate of Economic Depreciation = 5% year				
5	\$22.91	\$36.97	\$25.57	\$39.76
15	\$25.46	\$51.81	\$25.68	\$40.30
25	\$30.67	\$69.33	\$26.21	\$42.82
B. Expected Rate of Economic Depreciation = 15% per year				
5	\$96.51	\$155.56	\$94.99	\$152.77
15	\$96.76	\$160.10	\$94.99	\$152.77
25	\$98.47	\$171.86	\$94.99	\$152.98
C. Expected Rate of Economic Depreciation = 25% per year				
5	\$188.52	\$316.16	\$163.54	\$272.42
15	\$188.52	\$316.58	\$163.54	\$272.42
25	\$188.65	\$319.79	\$163.54	\$272.42

$T - K$. Table I presents insurance premiums appropriate for a five-year, five-payment lease and a seven-year, seven-payment lease with payments due at the beginning of each year. Columns 2 and 3 present premiums for a five-year lease which contains two and three cancellation options, respectively. That is, column 2 represents a five-year lease with cancellation options at the beginning of periods four and five, and column 3 represents a five-year lease with cancellation options at the beginning of periods three, four, and five. Similarly, columns 4 and 5 present premiums for a seven-year lease which contains two and three cancellation options, respectively. Panels A, B, and C of the table show premiums when the asset's expected rate of economic depreciation is 5 percent, 10 percent, and 15 percent per year, respectively. Finally, within each panel, rows 1, 2, and 3 give the premiums for asset variance rates of 5 percent, 10 percent, and 15 percent per year, respectively. (The variance rates are identified in column 1.)

The parameter values chosen for these examples are arbitrary, but they are not random. For example, Fama [11] has reported the standard deviations in monthly returns for 30 randomly selected companies listed on the New York Stock Exchange. When those estimates are converted to annualized variances, the range is 0.94 to 31.88 percent. The variance rates used in the examples approximately span that range. Similarly, the selected range of values for the expected rate of economic depreciation is likely to span the relevant range for the annual depreciation rates of most leased equipment. For example, an expected rate of depreciation of 15 percent per year implies that the expected salvage value of the asset will be slightly less than one-half its original value at the end of five years and will be approximately 20 percent of the original value at the end of 10 years.

The results given in the table illustrate that the size of the insurance premium is quite sensitive to the type of leased asset and the terms of the lease contract. For the five-payment lease, the insurance premiums range from as low as about 2 percent of the value of the leased asset (i.e., \$22.91) to as high as 32 percent of the value of the leased asset (i.e., \$319.79). For the seven-payment lease, the insurance premiums range from about 2½ to 27 percent of the initial value of the leased asset. (Keep in mind, of course, that these values represent the maximum insurance premium for a fully covered lease which contains no deductibles, no rebate, and no limit on the insurer's total liability.)

From the examples, it appears that the insurance premium is especially sensitive to changes in the expected rate of economic depreciation of the leased asset and to changes in the number of cancellation options contained in the lease. For example, a five-fold increase in the expected rate of economic depreciation is accompanied by a six- or seven-fold increase in the insurance premium, whereas a five-fold increase in the variance rate of change in the asset's value induces at most a two-fold increase in the insurance premium.

The table illustrates the indeterminancy in the sign of the partial derivative with respect to changes in the term-to-maturity of the lease. In most cases, the insurance premium under the seven-year lease is less than the premium under the five-year lease. The exceptions occur when very low expected rates of economic depreciation are coupled with low variance rates. In those cases, the insurance premiums under the five-year lease are less than those under the seven-year lease.

Table II focuses upon the effect of changes in the covariance term on the

Table II

Premiums for Cancellation Insurance Policy on Five-year Five-payment Lease: Sensitivity with Respect to Asset Variance Rate, Asset Rate of Economic Depreciation, and Asset Covariance Rate (Parameter Values: Initial Asset Cost= \$1,000; Risk-free Rate of Interest = 10% per year; and Two Cancellation Opportunities, $K = 3$)

Variance Rate of Change in Market Value of Leased Asset (%/yr)	Insurance Premiums		
	Covariance Rate = −0.25% per year	Covariance Rate = −0.5% per year	Covariance Rate = −1.0% per year
A. Expected Rate of Economic Depreciation = 5% per year			
5	\$24.29	\$25.65	\$28.50
15	\$26.75	\$28.02	\$30.72
25	\$31.87	\$33.15	\$35.68
B. Expected Rate of Economic Depreciation = 15% per year			
5	\$98.34	\$100.17	\$103.84
15	\$98.55	\$100.38	\$104.05
25	\$100.22	\$101.97	\$105.51
C. Expected Rate of Economic Depreciation = 25% per year			
5	\$190.32	\$192.10	\$195.64
15	\$190.32	\$192.10	\$195.64
25	\$190.44	\$192.18	\$195.72

insurance premium. The example uses a five-year, five-payment lease which contains two cancellation opportunities. Again, the initial asset value is assumed to be \$1,000, and the risk-free rate of interest is assumed to be 10 percent per year. Columns 2, 3, and 4 present premiums when the covariance rates are assumed to be −0.25 percent, −0.5 percent and −1.0 percent per year, respectively. As with the variance term, the values used for the covariance term were chosen so as to approximately span the range of common stock covariances as reported in Fama [11], with, of course, the terms having the opposite sign.

As the table indicates, the effect of a decrease in the (negative) covariance term is to increase the insurance premium. It is difficult to compare directly the effect of changes in the variance and covariance terms on the insurance premium. However, the table does indicate that at low expected rates of economic depreciation, the insurance premium is sensitive both to changes in the variance and the covariance terms. Contrarily, at high expected rates of economic depreciation of the leased asset, the insurance premium is relatively insensitive to changes in the variance term, but continues to be sensitive to changes in the covariance term. Nevertheless, the examples illustrate that changes in the expected rate of economic depreciation dominate changes in the other two variables. It should be noted, however, that this result is based on two cancellable lease payments. Increasing the cancellation opportunities (i.e., decreasing K) would have a further effect on the sensitivity of the insurance premium to changes in the variance and covariance terms.

As noted above, the partial derivative of the insurance premium with respect to changes in the risk-free rate is of indeterminant sign. Numerical examples illustrating the effect of changes in the risk-free rate on the insurance premium reveal some interesting patterns. These patterns are portrayed in Figure 1. As with the previous example, these numerical examples assume a five-year, five-payment lease with two cancellation opportunities. Panels A, B, and C present results for rates of economic depreciation of 5 percent, 15 percent, and 25 percent per year, respectively. Each panel contains three plots representing insurance premiums for assets with variance rates of 5 percent, 15 percent, and 25 percent per year.

According to Panel A, in which the expected rate of economic depreciation is 5 percent, when the variance rate is 5 percent per year, increases in the risk-free rate increase the insurance premium. When the variance rate is 25 percent per year, increases in the risk-free rate first increase and then decrease the insurance premium. When the variance rate is 15 percent per year, increases in the risk-free rate first increase, then decrease, then increase again the insurance premium.

According to Panel B, in which the rate of economic depreciation is 15 percent per year, when the asset's variance rate is 5 percent or 15 percent per year, increases in the risk-free rate first increase then decrease the insurance premium. When the variance rate is 25 percent per year, increases in the risk-free rate lead to decreases in the insurance premium.

Finally, in Panel C, in which the rate of economic depreciation is 25 percent per year, increases in the risk-free rate of interest lead to decreases in the insurance premium.

A. Expected Rate of Economic Depreciation = 5% per year

B. Expected Rate of Economic Depreciation = 15% per year

C. Expected Rate of Economic Depreciation = 25% per year

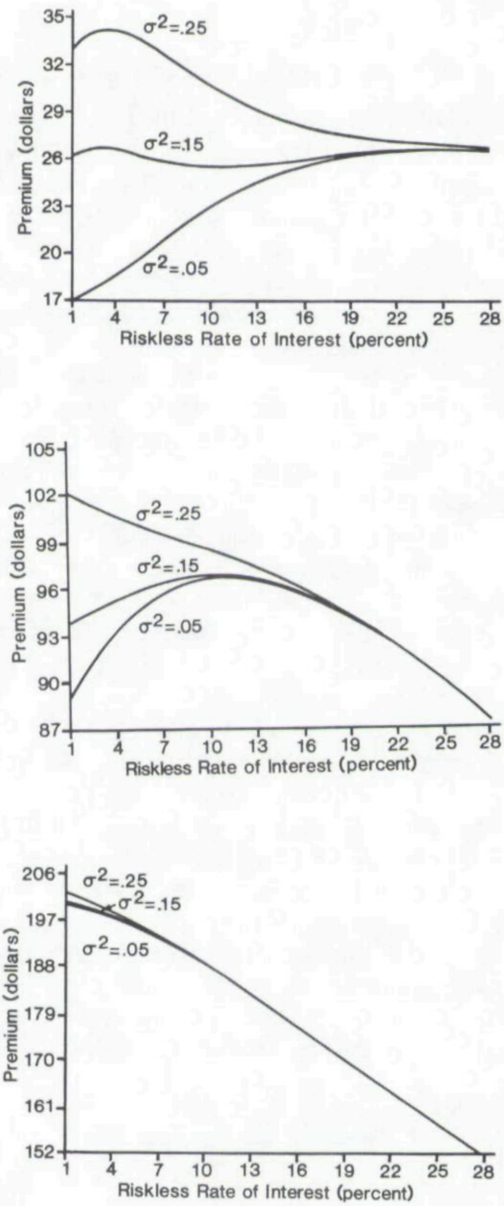

Figure 1. Plots of the Premiums for Cancellation Insurance Policy on Five-year Five-payment Lease: Sensitivity with Respect to Riskless Rate of Interest, Asset Variance Rate, and Asset Rate of Economic Depreciation (Parameter Values: Initial Asset Cost = \$1,000; Covariance Rate = 0; and Two Cancellation Opportunities, $K = 3$)

Interestingly, in all three panels, the three plots converge at "high" interest rates.[14]

V. The Problems of Moral Hazard and Adverse Selection

The equilibrium model described by Equation (7) was developed under the assumption of a perfectly competitive capital market in which information is costlessly available to all market participants. In the presence of costly information and information asymmetries, moral hazard and adverse selection problems may exist. The existence of these problems may complicate the equilibrium process and may have an impact upon the size of the fair insurance premium.

First, consider the moral hazard problem. Although this was not the case in the insurance contracts written by Lloyd's of London, in some cases the equipment lessors who offer cancellable lease contracts may also produce the leased asset. There then exists the potential for the lessor to affect the price of the asset by controlling the rate of technological innovation of the leased asset. If, after the insurance is purchased, the manufacturer/lessor introduces a new (more efficient) product, the effect will be to cause the value of existing leased assets to fall, thereby increasing the probability of lease cancellation.

Now, consider the problem of adverse selection. As with virtually all types of insurance, the potential for adverse selection exists under lease cancellation insurance policies.[15] Given the moral hazard problem that exists when the lessor controls the rate of product innovation, it is likely that those manufacturers/lessors who are most likely to introduce new products are also most likely to demand lease cancellation insurance. Similarly, the demand for cancellation insurance is likely to be greatest for those assets in which the probability of technological innovation is greatest.

Resolution (or at least partial resolution) of the moral hazard and adverse selection problems may come about due to the various risk-sharing provisions contained in lease cancellation insurance contracts. Holstrom [15, especially pp. 80–81], Mayers and Smith [18], Raviv [22], and Shavell [24, especially p. 66] describe the way in which the various risk-sharing provisions contained in insurance contracts serve to reduce the inevitable moral hazard and adverse selection problems that arise in insurance markets. In general, risk-sharing provisions include deductibles, upper limits on the insurer's liability, and various

[14] In footnote 9, we noted the analogy between lease cancellation insurance and an American put option. These numerical examples strengthen the analogy. At high levels of the risk-free rate, the premium declines with further increases in the risk-free rate regardless of the values taken on by the other parameters. This result corresponds with the finding by Geske and Johnson [14], that the partial derivative of the value of an American put option with respect to increases in the risk-free rate is strictly negative. Thus, at high levels of the risk-free rate of interest, the put option component of the cancellable lease dominates the fixed payment component.

[15] Gatto et al. [12] provide evidence that is consistent with the existence of adverse selection for one type of financial asset, namely mutual fund insurance. Adverse selection comes about because a single premium was charged for insurance on all types of mutual funds. As a consequence, owners of the riskier funds purchased mutual fund insurance proportionately more than owners of less risky funds. Like the results of Gatto et al. [12], our analysis suggests that a different premium be charged for different classes of assets and lease contracts.

forms of coinsurance. As we discussed in Section I, lease cancellation insurance contracts may contain each of these provisions. To the extent that these provisions reduce (or eliminate) problems introduced by moral hazard and adverse selection, the fair insurance premium derived under the assumptions of a perfectly competitive market with costlessly available information will provide a reasonable approximation to the full equilibrium insurance premium. To the extent that the various contract provisions are ineffective in resolving these problems, the competitive market insurance premium will provide a less accurate approximation. In either event, the simple model developed in this paper is properly viewed as the first step in the development of a complete model for the determination of fair premiums on lease cancellation insurance policies.

Even with this simple model, it is, however, necessary to assume that there exists a sufficiently large and diverse population of insurable leases such that the nonsystematic risk embodied in the individual contracts can be eliminated through diversification. That assumption is subject to eventual empirical examination. And, while we have not undertaken that investigation, it is reassuring to note that the leasing market has been estimated to encompass 20 percent of all corporate capital equipment acquisitions[16] and that the use of virtually all categories of capital equipment is available for acquisition by means of leasing contracts.[17]

VI. Conclusions

After a disastrous initiation and a near exodus, lease cancellation insurance has reemerged as a useful instrument in the financial marketplace. The model presented here could be used by insurers—including Lloyd's of London—in setting benchmarks to determine whether insurance premiums are sufficiently high to cover the risk exposure involved in their policies. As the numerical analyses demonstrate, the competitive premium is highly sensitive to the characteristics of the capital market, the characteristics of the leased asset, and the terms of the lease contract. We should emphasize that the model presented here is a simple one, and further embellishments could be made to enhance the realism of the model. For example, the model ignores taxes, and it assumes that the term structure of interest rates is known with certainty. Relaxing the first of these assumptions could conceivably have a significant effect on the size of the insurance premium. It is equally conceivable, however, that taxes could be shown—as they do in other well-known corporate financing problems—to have a neutral effect on the size of the competitive insurance premium. Relaxing the latter assumption is less likely to have a significant impact upon the amount of the competitive insurance premium. As a case in point, Brennan and Schwartz [3] demonstrate that the effect of a stochastic term structure on the pricing of convertible bonds is relatively slight in comparison with the impact of the risk associated with the underlying value of the firm.

The model also could be further embellished by explicitly incorporating other

[16] Brigham [6, p. 717].

[17] See, e.g., Sorenson and Johnson [25] and Crawford, Harper, and McConnell [9].

features of the insurance contract. For example, we have not explicitly taken into account the effect of the resale option, the total renewal liability, or the rebate provisions on the competitive premium. In general, each of these provisions tends to reduce the risk exposure of the insurer. As with the deductible provision—which appears to be a feature of most insurance contracts—the more restrictive these provisions are, the lower will be the insurance premium. Because policies can be tailor-made to fit specific lessors, there are undoubtedly other contract provisions or limitations that have been included in specific insurance contracts, but each of these would be specific to the particular contract in question. Rather than attempt to model each of those features, we have focused upon developing a model for pricing a simple policy that could serve as a starting point for more case-specific (and more complex) contracts.

Additionally, we have assumed away the problems of moral hazard and adverse selection which are present in virtually all insurance contracts. To the extent that the various risk-sharing provisions embodied in lease cancellation insurance policies do not eliminate these problems, our simple model represents only an approximation to a complete equilibrium solution and as a consequence, represents only the beginning of a more complete analysis of lease cancellation insurance policies.

REFERENCES

1. "Bad Luck Forces Updating at Lloyd's of London." *Business Week* (February 25, 1980), 94–108.
2. F. Black and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (May/June 1973), 637–59.
3. M. J. Brennan and E. S. Schwartz. "The Pricing of Equity-Linked Life Insurance Policies with an Asset Value Guarantee." *Journal of Financial Economics* 3 (June 1976), 195–213.
4. ———. "Alternative Investment Strategies for the Issuers of Equity-Linked Life Insurance Policies with an Asset Guarantee." *Journal of Business* 52 (January 1979) 63–94.
5. ———. "Analyzing Convertible Bonds." *Journal of Financial and Quantitative Analysis* 15 (November 1980), 907–30.
6. E. F. Brigham. *Fundamentals of Financial Management.* New York: The Dryden Press, 1982.
7. C. W. Cipolla and S. E. Spilka. "What Went Wrong at Lloyd's?" *Best's Review* 80 (November 1980), 22–26.
8. "Computer Lease Losses Exceeds Lloyd's Forecast." *The Journal of Commerce* (November 19, 1981), 7–8.
9. P. J. Crawford, C. P. Harper, and J. J. McConnell. "Further Evidence on the Terms of Financial Leases." *Financial Management* 10 (Autumn 1981), 7–14.
10. S. Emmrich. "Insuring Income: More Underwriters May Take the Risk." *Business Insurance* 13 (September 3, 1979), 1+.
11. E. F. Fama. *Foundations of Finance.* New York: Basic Books, Inc., 1976.
12. M. A. Gatto, R. Geske, R. Litzenberger, and H. Sossin. "Mutual Fund Insurance." *Journal of Financial Economics* 8 (September 1980), 283–317.
13. R. Geske. "The Valuation of Corporate Liabilities as Compound Options." *Journal of Financial and Quantitative Analysis* 12 (November 1977), 541–52.
14. ——— and H. E. Johnson. "The American Put Valued Analytically." *Journal of Finance* 39 (December 1984), 1511–24.
15. B. Holmstrom. "Moral Hazard and Observability." *Bell Journal of Economics and Management Science* 10 (Spring 1979), 74–91.
16. A. Kraus and S. A. Ross. "The Determination of Fair Profits for the Property-Liability Insurance Firm." *Journal of Finance* 37 (September 1982), 1015–28.
17. "Lloyd's Biggest Disaster." *Forbes* 123 (May 28, 1979), 38.

18. D. Mayers and C. W. Smith, "Contractual Provisions, Organizational Structure, and Conflict Control in Insurance Markets." *Journal of Business* 54 (July 1981), 407–34.
19. ——— and C. W. Smith. "On the Corporate Demand for Insurance." *Journal of Business* 55 (April 1982), 281–96.
20. J. J. McConnell and J. S. Schallheim. "Valuation of Asset Leasing Contracts." *Journal of Financial Economics* 12 (August 1983), 237–61.
21. M. H. Miller and C. W. Upton. "Leasing, Buying and the Cost of Capital Services." *Journal of Finance* 31 (June 1976), 141–83.
22. A. Raviv. "The Design of Optimal Insurance." *American Economic Review* 69 (March 1979), 84–96.
23. M. Rubinstein. "The Valuation of Uncertain Income Streams and the Pricing of Options." *Bell Journal of Economics and Management Science* 7 (Autumn 1976), 407–25.
24. S. Shavell. "Risk Sharing and Incentives in the Principal and Agent Relationship." *Bell Journal of Economics and Management Science* 10 (Spring 1979), 55–73.
25. I. Sorenson and R. Johnson. "Equipment Financial Leasing Practices and Costs: An Empirical Study." *Financial Management* 6 (Spring 1977), 33–40.
26. "Uncalculated Risk: At Lloyd's of London, A Record Loss Looms on Computer Policies." *Wall Street Journal* 194 (July 10, 1979), 1+.