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A Comparison of Alternative Models for Pricing GNMA Mortgage-Backed Securities

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I. Introduction

SINCE THE ISSUANCE OF the first Government National Mortgage Association (GNMA) mortgage-backed pass-through security in February 1970, the total amount of GNMAs issued has grown to over $105 billion. In terms of their trading volume, GNMA securities are the most actively-traded class of long-term fixed-rate instruments. However, this gross volume statistic masks the fact that GNMAs are not homogeneous securities. They differ by coupon interest rate and remaining term to maturity. There is also a widely-held belief [12] and some evidence [7] that GNMAs differ according to their expected prepayment rates.

The bulk of the trading volume in GNMAs is comprised of newly-issued securities. New securities are generally issued at the Federal Housing Administration (FHA) maximum interest rate and have terms to maturity of 30 years. While most GNMA trading is comprised of new issues, the bulk of the outstanding securities is comprised of "old" securities whose coupon rates may differ from the current FHA ceiling and whose remaining terms to maturity are less than 30 years. This raises a pricing problem for GNMA security dealers, portfolio managers, financial institutions that hold large portfolios of GNMA securities, and other potential GNMA investors. Up-to-date market price quotes are available for new issues, but quotes for "old" ones are not so readily available. Thus, potential traders confront the problem of pricing these infrequently traded securities.

The problem of pricing GNMA securities was initially addressed by Curley and Guttentag [5]. They presented a model for the pricing of GNMAs that was an imaginative and important extension of the then widely-used average life procedure. The particular innovation of their model was to incorporate estimates of the prepayment probabilities to determine expected future cash flows. Through simulation and sensitivity analysis, Curley and Guttentag (hereafter C & G) compared prices generated by their model with those generated by the traditional average life procedure.

In his discussion of the C & G paper, Brealey [1] encouraged the authors to extend their model to incorporate uncertainty and to value explicitly the call options attached to the underlying mortgage loans. In a previous paper [6], we

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took those additional steps by developing a pricing model for GNMA securities based on the general model for pricing interest contingent claims developed by Brennan and Schwartz [2] and Cox, Ingersoll, and Ross [3].

Still unanswered, however, is the question of whether there is an important difference in the prices generated by the alternative models. This paper is an attempt to fill that void. Proceeding in a fashion similar to C & G, we use simulation and sensitivity analysis to compare prices generated by the contingent claims model with prices generated by a variation of the C & G method and with those generated by the average life procedure. The rationale for these comparisons is that if there is not much difference in the prices generated by the alternative models, traders can safely use the simplest model for most purposes. On the other hand, if the prices generated by the models differ significantly in some or most circumstances, traders may wish to consider the more complex and (potentially) more refined models.

In Section II of the paper we briefly describe the GNMA security and in Section III we describe the alternative models for pricing GNMA securities. In Section IV we compare the prices generated by the three models under different assumptions about the economic environment. The last section contains a conclusion.

II. Characteristics of GNMA Mortgage-Backed Pass-Through Securities

GNMA mortgage-backed pass-through securities are issued by FHA-approved mortgages. GNMA requires that all of the individual loans which back a security have the same coupon interest rate and original term to maturity and that each be insured by the FHA or guaranteed by the Veterans Administration (VA).

The mortgage loans which back GNMA securities are fully amortizing. Each month the issuer of a GNMA security must "pass through" the scheduled interest and principal payments on the underlying mortgage loans to the holder of the security, whether or not the issuer has actually collected those payments from the individual mortgagors. The issuer must also pass through any additional amounts which are received from the mortgagors for loan prepayments (i.e., unscheduled principal repayments) and/or from the FHA or VA for settlements on those loans in the pool which have been foreclosed. Because GNMA monitors the performance of the security issuers and because the securities are backed by the "full faith and credit" of the U.S. Treasury, GNMA pass-through securities are generally considered to be default-free.

All FHA and VA mortgage loans can be prepaid (i.e., called by the mortgagor) at any time without a prepayment penalty (i.e., without the payment of a call premium). Furthermore, the loans are assumable. That is, the mortgagor may transfer his obligation for the debt. Hence, with FHA and VA mortage loans, there are not any contractual restrictions which limit mortgagors' call strategies. Rational mortgagors will adopt the policy which maximizes their wealth. Thus, when markets are frictionless, a mortgagor will exercise his call option whenever his existing loan can be refinanced with an otherwise identical loan which has a lower effective rate of interest than the rate on the existing loan. On the other hand, he will never prepay his loan when the market contract interest rate exceeds the contract rate of his current loan (i.e., when the market value of the existing loan is less than its remaining principal balance). We refer to this as the optimal call policy.

One of the notable characteristics of mortgagors is that, in practice, many of them call their loans even when the market interest rate is above the contract rate on their existing loans. These prepayments (which generally occur when a mortgagor changes his residence and the existing mortgage loan is not assumed by the purchaser of his house) are a constrained maximum from the perspective of a rational mortgagor. Nevertheless, we *refer* to these as "suboptimal" prepayments in order to distinguish them from optimal calls.

III. Alternative Models for Pricing GNMA Mortgage-Backed Securities

Much of the concern about pricing mortgage-backed securities has focused on the propensity of mortgagors to prepay or call their loans prior to maturity. In this section we describe the ways in which (1) the traditional "average life" method, (2) an abbreviated version of the C & G model (hereafter CG* model, where the asterisk indicates that we are using an abbreviated version of their model), and (3) the interest contingent claims model can be used to price GNMA securities. Each of these models takes a different approach to incorporating the effect of prepayments on the value of a GNMA security. Our description of the way in which the first two models can be used is based on conversations with security dealers.

III. A. The "Average Life" Model

The traditional average life procedure attempts to incorporate prepayments by assuming that the scheduled interest and principal will be paid on the loan until a period equal to the "average life" of a portfolio of "comparable mortgages." At that point, the remaining principal balance of the loan is assumed to be repaid in full. The average life can be estimated from FHA actuarial data for mortgage loan terminations.[1] In practice, however, the avereage life is typically assumed to equal 12 years. Given this assumed stream of cash flows and the current market price for a newly-issued GNMA, a yield-to-average life is computed for the newly-issued security.

An old security (i.e., one with a shorter remaining term to maturity than the new one) is priced by assuming that the yield on all outstanding GNMAs is the same as the yield on the newly-issued one. The expected average life of an old security is assumed to equal the average life of a comparable portfolio of mortgage loans which have the same remaining terms to maturity as the security to be priced. At the end of a period equal to its expected average life, the loan is assumed to be repaid in full. The estimated price of an old security is obtained by discounting the assumed stream of cash flows to the present at the computed yield on the newly-issued security.

[1] See Curley and Guttentag [4] for a thorough description of these data.

III. B. The CG* Model

C & G correctly criticize the average-life procedure for ignoring the potential for a premature loan repayment either before or after the expected average life. C & G correct for this deficiency by incorporating the entire time distribution of prepayment probabilities into the yield calculation. They used linear regression and FHA historical data to estimate the conditional probabilities of a loan prepayment at the end of each month of the loan's life given that it is outstanding at the beginning of the month.[2]

A variation of the C & G method, which we label CG*, that appears to have been widely adopted (see, for example, [11]) requires that the yield-to-maturity on a newly-issued security be computed by solving the following equation for y:

$$M_t = \sum_{j=t+1}^{t+n} \left((C_j + P_j F_j) e^{y(t-j)} \right) \left(\prod_{k=1}^{j-t} (1 - P_{j-k}) \right) \tag{1}$$

where M_t is the market value of a loan after it has been outstanding for t months; C_j is the scheduled principal and interest payment on the loan at the end of month $j-1$; F_j is the scheduled remaining principal of the security at the end of month $j-1$; and P_{j-k} is the conditional probability that the loan will be prepaid at the end of month $j-k-1$, given that it is outstanding at the beginning of the month (P_{j-k} is equal to zero for $j-k=t$); and n is the number of remaining scheduled cash flows. The conditional probabilities of a loan prepayment are computed from FHA actuarial data.

The yield-to-maturity computed with equation (1) for a newly-issued security is used to price old securities by assuming that the yield-to-maturity is the same for all GNMAs. Given this yield, equation (1) is solved for the value of other GNMA securities with shorter remaining terms to maturity.

III. C. The Contingent Claims Model

In a previous paper [6], we developed a model for valuing GNMA securities which is based on a general equilibrium theory of the term structure of interest rates under uncertainty derived by Cox, Ingersoll, and Ross [3]. We consider an economy where the current interest rate for instantaneous riskless borrowing and lending completely summarizes all information relevant for pricing default-free, fixed-interest rate securities. The risk-free interest rate is assumed to follow the mean reverting stationary Markov process given by the stochastic differential equation

$$dr = b(r)\,dt + s(r)\,dz, \tag{2}$$

where

$$b(r) \equiv k(m-r), \quad k, m > 0,$$
$$s(r) \equiv s\sqrt{r}, \quad s \text{ constant},$$

and dz is a Wiener process. The function $b(r)$ is the instantaneous expected change in the interest rate; k is the speed of adjustment parameter; m is the

[2] The exact estimation procedure is described in [4].

steady-state mean of the process; and the function $s(r)^2$ is the instantaneous variance. Negative interest rates are precluded with this mean reverting interest rate process and the variance of the process increases with the interest rate.

When a mortgagor makes a suboptimal prepayment, investors receive the principal balance remaining at that time; hence, the market value of the security "jumps" to its remaining principal balance and the security ceases to exist. We model suboptimal prepayments as a Poisson-driven or jump process.[3] Let the random variable y equal 0 if the loan has not been called and equal 1 if it has been called. The Poisson process, dy, is given by

$$dy = \begin{cases} 0 \text{ if a suboptimal prepayment does not occur} \\ 1 \text{ if a suboptimal prepayment occurs} \end{cases} \tag{3}$$

where

$$E(dy) = \lambda(r, \tau)dt,$$

and $\lambda(r, \tau)$ is the probability per unit of time of a suboptimal prepayment at time to maturity τ and interest rate r.

Given the stochastic processes of the current interest rate and the suboptimal prepayments, it follows that the value of a GNMA security, $V(r, y, \tau)$, which is a function of the two state variables, r and y, and its remaining term to maturity, τ, is governed by the mixed process

$$dV = [a(r, \tau)V - C(\tau) - \lambda(r, \tau)(F(\tau) - V)]dt + h(r, \tau)Vdz + [F(\tau) - V]dy. \tag{4}$$

In (4), $a(r, \tau)$ is the instantaneous expected rate of return on the security, $h(r, \tau)$ is the instantaneous standard deviation of the return conditional on the Poisson event not occurring, and $C(\tau)$ and $F(\tau)$ are the scheduled cash flow (per unit of time) and remaining principal balance, respectively, at time to maturity τ. From Ito's lemma and an analogous lemma for Poisson processes, we obtain

$$a(r, \tau) = [\tfrac{1}{2}s(r)^2V_{rr} + b(r)V_r - V_\tau + C(\tau) + \lambda(r, \tau)(F(\tau) - V)]/V \tag{5}$$

and

$$h(r, \tau) = s(r)V_r/V,$$

where subscripts on V denote partial derivatives.

In [6] we show that if the risk associated with suboptimal prepayments is diversifiable, a GNMA security must, to avoid arbitrage opportunities, be priced so that its expected return equals the riskless interest rate plus a risk premium. That is,

$$a(r, \tau) = r + p(r)h(r, \tau), \tag{6}$$

where $p(r)$ is the price of interest rate risk for all interest-dependent securities. Because V_r is generally negative, $h(r, \tau)$ is negative, and the risk premium is positive when $p(r)$ is negative.

[3] Ingersoll [8] and Merton [10] have used this approach previously to deal with similar problems.

The partial differential equation for the value of a GNMA security is obtained by substituting from (5) for $a(r, \tau)$ and $h(r, \tau)$ in (6). Making these substitutions and rearranging yields

$$\tfrac{1}{2}s(r)^2V_{rr} + [b(r) - p(r)s(r)]V_r - V_\tau + \lambda(r, \tau)[F(\tau) - V] + C(\tau) = rV. \qquad (7)$$

The combination of the first three terms on the left-hand side of (7) is the expected (conditional on the information set at time to maturity τ) risk-adjusted price change of a GNMA security given that a suboptimal prepayment does not occur. The last two terms on the left-hand side of (7) are the expected change in the value of the security from a suboptimal prepayment and the scheduled cash flow, $C(\tau)$, respectively, when the remaining time to maturity is τ and the current interest rate is r. These terms are not risk-adjusted because $C(\tau)$ and $F(\tau)$ are default-free and because there is not a risk premium associated with the suboptimal prepayments. Hence, (7), like (6), requires that the expected risk-adjusted return on a GNMA security be equal to the risk-free return.

We assume that the risk adjustment term, $p(r)s(r)$, in (7) is proportional to the current interest rate, i.e., $p(r)s(r) = qr$, where q is the proportionality factor. Making this substitution and substituting from (2) for $b(r)$ and $s(r)$ yields:

$$\tfrac{1}{2}s^2rV_{rr} + [km - (k + q)r]V_r - V_\tau - rV + \lambda(r, \tau)[F(\tau) - V] + C(\tau) = 0. \qquad (8)$$

With the initial condition that the value of an amortizing security is zero at maturity and the boundary conditions that (1) the value of the security goes to zero as the interest rate approaches infinity and (2) the value of the security can never exceed its remaining principal balance, equation (8) can be solved numerically for the value of a GNMA security. The latter boundary condition is deduced from the effect of the optimal call policy. With frictionless markets, mortgagors will not allow the market value of a callable loan to exceed its remaining principal balance because, if it did, they could refinance their existing loans at lower coupon interest rates. Thus, for each τ there is some level of the riskless interest rate at which the call option will be exercised. Riskless interest rates below that level are not relevant for pricing GNMA securities.

IV. Comparison of the Alternative Models

Before presenting the results of the simulations, some discussion of the differences among the alternative models is appropriate. The yield-based models are discrete-time, static, certainty models which assume an unchanging, flat term structure. On the other hand, the contingent claims model is a continuous-time, dynamic, uncertainty model which is based on a general equilibrium theory of the term structure. All three models value GNMAs as single default-free mortgage loans.

The continuous time analogs (i.e., the summation in (1) is replaced with integration) of the average life and CG* models are special cases of the contingent claims model which can be obtained by imposing restrictions on the contingent claims model. In particular, the first term on the left-hand side of (8) is zero under certainty and the second term is zero if we additionally assume a flat term structure. The loan would never be called optimally under those assumptions and the yield-based models incorporate suboptimal prepayments by replacing the scheduled cash flows with future cash flow patterns that are assumed to be known with certainty. With the CG* model, an amortization rate that is more rapid than the scheduled rate is posited. With the average life model, the security is assumed to have a final balloon payment and a shorter term to maturity than its scheduled term to maturity. Hence, with the cash flows assumed by the yield-based models, the term $\lambda(r, \tau)[F(\tau) - V]$ drops out of (8) and the scheduled rate of cash flow, $C(\tau)$, is replaced by the assumed rate of cash flow. Under these assumptions (8) degenerates to an ordinary differential equation: the solution to that equation is the continuous time analog of (1). Thus, when the above assumptions provide a good approximation of the economic environment, the differences among the prices obtained from the three models will be small. The simulations presented here are designed to determine if the models give substantially different prices under alternative plausible scenarios of the market environment. In particular, we compare the prices generated by the three models when the current term structure is ascending and when it is descending in a market environment where future term structures and prepayment cash flows are uncertain.

To compare prices generated by the three models, we use the contingent claims model to generate the price of a newly-issued security (i.e., a security with a 30-year term to maturity) that is expected to incur suboptimal prepayments at a specified percentage of the FHA historical rate. We then use that price as the benchmark price in order to compute yields-to-maturity by means of the average life procedure and the CG* model. These yields are then used, as described in Sections III.A and III.B, to price "old" GNMAs. These prices are then compared to the prices of "old" securities generated with the contingent claims model.[4]

For the numerical solutions presented, we assume that the mean of the current interest rate, m, is .056; the variance of the current interest rate, s^2, is .008; the speed of adjustment parameter, k, is .8; and the risk adjustment parameter, q, is $-.247$.[5] When $k = .8$, the current interest rate is expected to revert halfway back to its mean in 10.4 months. With these parameters the term structure of interest rates has a natural tendency to be ascending and, regardless of the value of current interest rate, the term structure approaches a long-run interest rate, $R(\infty)$, of .08 per year as time approaches infinity.

For the model of the term structure assumed in this paper, when the current interest rate, r, is below the long-run interest rate, $R(\infty)$, the term structure is upward sloping. The term structure is humped when r is between $R(\infty)$ and $km/(k + q)$, and it is downward sloping when r is above $km/(k + q)$. In Table 1 we

[4] We should emphasize that using the assumed set of term structure parameters and the contingent claims model to generate the benchmark price of the newly-issued security and then using those same parameters to price "old" securities with the same model does not work to the disadvantage of the average life and CG* models. In fact, when the cash flows assumed by the average life and CG* models are discounted at the interest rates given by the term structure, the resulting price differences are, in general, substantially larger than those shown. This is because the interest rates given by the term structure are not appropriate for discounting the cash flows from callable securities.

[5] These parameters are similar to those estimated by Ingersoll [9].

set r equal to .06 to compare the prices generated by the models when the term structure is rising. In Table 2 we set r equal to .12 to compare the prices when the term structure is falling. In panel A of each table, we assume that the expected prepayment rate is 100 percent of the FHA historical average. In panels B and C, the expected prepayment rates are 200 percent and 300 percent, respectively, of the FHA historical average.[6] Finally, in each panel we present prices for GNMAs with coupon interest rates of 6.5 percent and 8 percent.

Columns 2, 3, and 4 of each table contain the prices generated by the average life model, the CG* model, and the contingent claims model, respectively, when the security's coupon interest rate is 6.5 percent.[7] Column 5 (6) shows the difference between the prices of the average life model and the contingent claims model (the CG* model and the contingent claims model) when the coupon interest rate is 6.5 percent. Columns 7, 8, and 9 contain the prices generated by the three models when the security's coupon interest rate is 8 percent. Columns 10 and 11 show the differences between the prices generated by the contingent claims model and each of the other models for 8 percent securities.

Several general conclusions can be drawn from the tables. The closer the newly-issued security's price is to par (i.e., 100.00), the smaller the differences between the prices (for "old" securities) generated by the alternative models. This is due to several characteristics of the security's value. First, the closer the security's price to par, the less sensitive is the price to changes in the interest rate and, therefore, uncertainty has a smaller impact on the security's value. This is because the value of the call option is greatest when the security's price is close to par. Further, when the security's price is close to par, the value of a suboptimal prepayment is small and the value of the security is less sensitive to alternative assumptions about the stream of cash flows. Finally, because the value of the security approaches par as the remaining term to maturity decreases, the yields computed for new and old securities (using prices obtained from the contingent claims model) are nearly identical when the value of the newly-issued security is close to par.

Another systematic pattern which is revealed by the tables is that when the remaining term to maturity is long (i.e., greater than 25 years), the prices obtained with the average life model are always less than the prices obtained with the contingent claims model and they are generally less than those obtained with the CG* model. The price differences are larger the further the price of the newly-issued security is from par and the higher the expected prepayment rate. This is because the expected prepayment rates increase dramatically in the first five years after the security's issuance and, therefore, the prices generated with the contingent claims and the CG* models increase rapidly during those years. However, the prices obtained with the average life procedure, which ignores the

[6] The expected prepayment rates are computed from the FHA survival rates in [11]. Based on those data it is not possible to estimate $\lambda(r, \tau)$ as a function of both r and τ. Hence, for Tables 1 and 2 we take λ to be a function of τ only.

[7] We use "average life" as a generic name for any procedure which assumes the loans will be prepaid in full at a specified point in time. For the prices shown in the tables, average life is computed as $\min[12, \tau/2]$.

Table 1

Comparison of Alternative Models for Pricing GNMA Mortgage-Backed Securities when the Current Interest Rate is 6%—Term Structure is Rising

	Coupon Interest Rate is 6½%					Coupon Interest Rate is 8%				
(1) Term to Maturity	(2) Average Life Model	(3) C & G* Model	(4) Contingent Claims Model	(5) Difference (4) − (2)	(6) Difference (4) − (3)	(7) Average Life Model	(8) C & G* Model	(9) Contingent Claims Model	(10) Difference (9) − (7)	(11) Difference (9) − (8)
A. Expected Prepayment Rate Equals 100 Percent of FHA Experience										
30	92.51420	92.51416	92.51407			99.62109	99.62108	99.62101		
25	92.75493	93.63837	93.88351	1.12857	0.24514	99.63200	99.67667	99.74003	0.10802	0.06336
20	93.74492	94.33360	94.78113	1.03621	0.44753	99.68102	99.71098	99.77596	0.09494	0.06497
15	95.03840	95.39887	96.09719	1.05879	0.69832	99.74619	99.76471	99.85084	0.10466	0.08614
10	96.49960	96.61989	97.48287	0.98328	0.86298	99.82060	99.82691	99.95893	0.13833	0.13202
5	98.13241	97.96617	98.84914	0.71673	0.88297	99.90425	99.89588	99.99400	0.08975	0.09812
B. Expected Prepayment Rate Equals 200 Percent of FHA Experience										
30	94.51125	94.51123	94.51118			99.75939	99.75930	99.75924		
25	94.68891	95.60406	95.85277	1.16386	0.24871	99.76632	99.80649	99.85878	0.09246	0.05229
20	95.42424	95.99292	96.37152	0.94729	0.37861	99.79748	99.82304	99.88225	0.08477	0.05921
15	96.38007	96.74819	97.32024	0.94017	0.57205	99.83888	99.85599	99.95199	0.11311	0.09601
10	97.45327	97.60996	98.29492	0.84165	0.68497	99.88614	99.89395	99.98556	0.09941	0.09160
5	98.64520	98.46019	99.16005	0.51485	0.69986	99.93925	99.93164	100.00000	0.06075	0.06836
C. Expected Prepayment Rate Equals 300 Percent of FHA Experience										
30	95.68278	95.68270	95.68265			99.82466	99.82461	99.82456		
25	95.82304	96.73566	96.98739	1.16435	0.25173	99.82972	99.86693	99.94035	0.11063	0.07342
20	96.40572	96.96788	97.29853	0.89281	0.33065	99.85243	99.87603	99.95707	0.10464	0.08104
15	97.16089	97.53694	98.02037	0.85948	0.48343	99.88261	99.89901	99.98169	0.09908	0.08268
10	98.00582	98.20112	98.76900	0.76319	0.56789	99.91705	99.92610	99.99381	0.07676	0.06772
5	98.94092	98.78721	99.35975	0.41882	0.57254	99.95574	99.95013	100.00000	0.04426	0.04987

Parameters of interest rate process: $m = .056$; $k = .8$; $s^2 = .008$; $q = -.24714$; $R(\infty) = .08$.

Table 2

Comparison of Alternative Models for Pricing GNMA Mortgage-Backed Securities when the Current Interest Rate is 12%—Term Structure is Falling

	Coupon Interest Rate is 6½					Coupon Interest Rate is 8%				
(1) Term to Maturity	(2) Average Life Model	(3) C & G* Model	(4) Contingent Claims Model	(5) Difference (4) − (2)	(6) Difference (4) − (3)	(7) Average Life Model	(8) C & G* Model	(9) Contingent Claims Model	(10) Difference (9) − (7)	(11) Difference (9) − (8)
A. Expected Prepayment Rate Equals 100 Percent of FHA Experience										
30	84.61329	84.61326	84.61323			92.49567	92.49556	92.49552		
25	85.09442	86.81383	86.35128	1.25686	−0.46255	92.70671	93.55165	93.06893	0.36221	−0.48273
20	87.01961	88.14478	87.26999	0.25037	−0.87479	93.63154	94.19079	93.25891	−0.37263	−0.93187
15	89.58996	90.26373	88.81166	−0.77830	−1.45207	94.88452	95.22492	93.74542	−1.13911	−1.47951
10	92.56950	92.76056	90.80922	−1.76028	−1.95134	96.34778	96.45103	94.55234	−1.79544	−1.89869
5	95.98646	95.58273	93.38270	−2.60376	−2.20003	98.03000	97.83973	95.79050	−2.23949	−2.04923
B. Expected Prepayment Rate Equals 200 Percent of FHA Experience										
30	86.69454	86.69456	86.69449			93.01320	93.01315	93.01308		
25	87.11383	89.25680	88.76155	1.64772	−0.49525	93.21005	94.34615	93.87892	0.66887	−0.46723
20	88.80429	90.12753	89.33194	0.52766	−0.79558	94.07430	94.79505	94.02380	−0.05050	−0.77125
15	91.04794	91.89932	90.62247	−0.42547	−1.27685	95.24353	95.72565	94.51423	−0.72930	−1.21142
10	93.63044	93.97964	92.36350	−1.26695	−1.61615	96.60666	96.82366	95.32201	−1.28465	−1.50165
5	96.57108	96.07205	94.31970	−2.25138	−1.75235	98.17111	97.93085	96.33231	−1.83880	−1.59854
C. Expected Prepayment Rate Equals 300 Percent of FHA Experience										
30	88.01417	88.01422	88.01413			93.41106	93.41106	93.41103		
25	88.39369	90.86215	90.34052	1.94683	−0.52163	93.59696	94.96752	94.50138	0.90442	−0.46614
20	89.93091	91.45430	90.71540	0.78448	−0.73890	94.41431	95.28621	94.60787	0.19357	−0.67834
15	91.96373	92.98347	91.84153	−0.12220	−1.14195	95.51888	96.12706	95.07863	−0.44024	−1.04843
10	94.29333	94.82207	93.43540	−0.85793	−1.38668	96.80495	97.14202	95.88675	−0.91819	−1.25527
5	96.93433	96.47014	95.02211	−1.91223	−1.44804	98.27903	98.05381	96.74750	−1.53152	−1.30630

Parameters of interest rate process: $m = .056$; $k = .8$; $s^2 = .008$; $q = -.24714$; $R(\infty) = .08$.

possibility of a prepayment in the early years of the security's life, approach an asymptote as the term to maturity is lengthened to 30 years.

Table 1 shows that when the current interest rate is .06, the average life procedure and the CG* model consistently give prices that are less than those generated by the contingent claims model. Conversely, Table 2 shows that when the current interest rate is .12, the CG* model consistently yields prices that are greater than those generated by the contingent claims model and the average life procedure gives prices which are greater than those obtained from the contingent claims model when the remaining term to maturity is less than 20 years. These results are due to the assumption, embodied in the average life procedure and the CG* model, that the term structure is flat. Thus, all future cash flows are discounted at the same interest rate. When the term structure is rising, this procedure will "overdiscount" near-term cash flows and it will "underdiscount" far-term cash flows. The reverse will be true when the term structure is downward sloping. Contrarily, the contingent claims model makes use of information contained in the entire term structure when discounting future cash flows. When the term structure is flat, the prices generated by the three models are nearly identical.

The effect of the coupon interest rate can be seen by comparing the differences in prices for coupon rates of 6.5 percent (columns 5 and 6) and 8 percent (columns 10 and 11). When the term structure is rising (Table 1), the differences between the prices generated by the models are greater for securities with lower coupon rates than for those with higher coupon rates. This is true regardless of the remaining term to maturity of the securities. When the term structure is falling (Table 2), the differences between the prices generated by the alternative models are, in general, greater for securities with lower coupon rates than for those with higher coupon rates. Comparing the prices from the CG* and contingent claims models, the differences are always greater for the security with the lower coupon rate when the expected prepayment rate equals either 200 or 300 percent of FHA experience. However, when the expected prepayment rate equals 100 percent of FHA experience, the differences are smaller (larger) for the 6.5 percent security than the 8 percent security when the remaining term to maturity is long (short). We should note, in this regard, that the difference in prices generated by the alternative models is zero for securities with 30 years to maturity and for those with zero years to maturity. Thus, as the term to maturity declines, the absolute value of the differences in prices given by the CG* and contingent claims models first increase and then decrease. The rate at which this phenomenon occurs depends upon the characteristics of the security and the value taken on by the parameters of the term structure.[8]

[8] The results shown here are representative of those obtained with other assumptions about the parameters of the interest rate process. In particular, the prices obtained from the CG* model are, in general, greater than, nearly equal to, or less than those obtained from the contingent claims model when the term structure is descending, nearly flat, or ascending. In general, the smallest differences are obtained when the value of the newly-issued security is close to par. Hence, for a given deviation of the current interest rate from the long-run interest rate, $R(\infty)$, the differences are generally smaller when the term structure is rising than when it is falling. As the mean of the current interest rate, m, is increased, holding k, s^2 and q constant, the value of the newly-issued security decreases and the differences in the prices obtained from the alternative models are much larger than those shown in Table 1.

Table 3

Comparison of Alternative Models for Pricing 8 Percent GNMA Mortgage-Backed Securities when the Current Interest Rate is 12 Percent and the Mean of the Poisson Prepayment Process is a Function of the Current Interest Rate and Time to Maturity

	$\lambda(r, \tau) = P(\tau)e^{10(\min(0,\ .08-r))}$					$\lambda(r, \tau) = P(\tau)e^{25(\min(0,\ .08-r))}$				
(1) Term to Maturity	(2) Average Life Model	(3) C & G* Model	(4) Contingent Claims Model	(5) Difference (4) − (2)	(6) Difference (4) − (3)	(7) Average Life Model	(8) C & G* Model	(9) Contingent Claims Model	(10) Difference (9) − (7)	(11) Difference (9) − (8)
A. Expected Prepayment Rate Equals 100 Percent of FHA Experience										
30	92.33100	92.33093	92.33087			92.17281	92.17281	92.17273		
25	92.54655	93.40910	92.79732	0.25078	−0.61178	92.39267	93.27215	92.53495	0.14228	−0.73720
20	93.49054	94.06127	93.00351	−0.48703	−1.05775	93.35504	93.93679	92.75924	−0.59580	−1.17756
15	94.77009	95.11734	93.47458	−1.29550	−1.64276	94.66006	95.01390	93.21396	−1.44610	−1.79994
10	96.26518	96.37018	94.26650	−1.99868	−2.10369	96.18572	96.29241	93.99017	−2.19555	−2.30224
5	97.98492	97.78988	95.58495	−2.39997	−2.20493	97.94154	97.74190	95.39043	−2.55111	−2.35146
B. Expected Prepayment Rate Equals 200 Percent of FHA Experience										
30	92.76344	92.76340	92.76332			92.51472	92.51470	92.51463		
25	92.96715	94.14260	93.45221	0.48506	−0.69039	92.72523	93.93980	93.02389	0.29866	−0.91591
20	93.86069	94.60633	93.61740	−0.24329	−0.98893	93.64784	94.41820	93.20877	−0.43907	−1.20943
15	95.07039	95.56914	94.09989	−0.97050	−1.46925	94.89775	95.41303	93.67390	−1.22385	−1.73913
10	96.48186	96.70620	94.88650	−1.59536	−1.81970	96.35733	96.58895	94.43530	−1.92202	−2.15365
5	98.10311	97.85347	96.00577	−2.09734	−1.84770	98.03520	97.77616	95.67424	−2.36096	−2.10192
C. Expected Prepayment Rate Equals 300 Percent of FHA Experience										
30	93.09816	93.09819	93.09811			92.77701	92.77698	92.77689		
25	93.29268	94.72682	93.97412	0.68145	−0.75269	92.98034	94.47956	93.42068	0.44034	−1.05888
20	94.14693	95.05944	94.10778	−0.03915	−0.95166	93.87230	94.82638	93.58132	−0.29098	−1.24506
15	95.30237	95.93901	94.57907	−0.72330	−1.35994	95.07980	95.74555	94.04398	−1.03582	−1.70157
10	96.64906	97.00199	95.36904	−1.28001	−1.63295	96.48865	96.85781	94.79917	−1.68948	−2.05863
5	98.19419	97.95751	96.34799	−1.84619	−1.60952	98.10681	97.85828	95.91729	−2.18952	−1.94099

Parameters of interest rate process: $m = .056$, $k = .8$, $s^2 = .008$, $q = -.24714$, $R(\infty) = .08$.

In the previous comparisons we assumed, for all levels of the current interest rate, that the mean, $\lambda(r, \tau)$, of the Poisson process driving the suboptimal prepayments equaled the prepayment probabilities, $P(\tau)$, used by the CG* model. However, Curley and Guttentag [4] provide empirical evidence that the prepayment probabilities decrease as the current interest rate increases. We incorporate that effect by allowing the prepayment probabilities to depend on both the current interest rate and the remaining time to maturity. Specifically, for each remaining time to maturity, we assume that the prepayment probabilities decay exponentially as the riskless interest rate at that time rises above the security's coupon rate. In Table 3 we compare the prices obtained from the alternative models for an 8 percent GNMA when the current interest rate is 12 percent. The rate of decay assumed for the comparisons shown on the left-hand side of the table is smaller than that assumed on the right-hand side of the table. Comparing the differences shown in Table 2 for an 8 percent security with those shown in Table 3 indicates that the absolute value of the differences in prices obtained from the contingent claims model and the CG* model increase as the rate of decay increases and that this effect is larger the higher the expected prepayment rate.[9] This is because the CG* model assumes the prepayments are certain, regardless of the level of future interest rates.

V. Conclusion

In this paper we use simulation and sensitivity analysis to compare alternative models for the pricing of GNMA mortgage-backed securities. We compare prices generated with a contingent claims model developed in [6] to those generated by the traditional average life procedure and a variation of a model developed by Curley and Guttentag [5]. The bottom line in all of these comparisons is whether the differences in prices are "significant." In a large measure, the answer to that question lies in the eye of the beholder. However, prices for GNMAs are quoted in intervals of one thirty-second of a point. If we use one thirty-second of a dollar as the benchmark, all of the differences in prices shown are significant. Given the large quantities in which most institutions trade GNMA securities, small differences in prices represent large dollar amounts. The remaining empirical question is whether the contingent claims model yields "good" predictions of observed market prices. If so, the contingent claims model should be useful to GNMA dealers and the portfolio managers of savings banks, life insurance companies, and pension funds who are the primary traders of GNMA mortgage-backed securities.

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[9] The comparisons shown here cast the yield-based models in their best light because to compute the prices using the average life procedure and the CG* model, we assumed that a "market" price for a newly-issued security was available for each coupon rate and expected prepayment rate considered. For example, if we had used the price of an 8 percent GNMA with an expected prepayment rate equal to 100 percent of FHA experience as the basis for pricing another GNMA with a different coupon rate or a different prepayment rate, the differences would be larger than those shown in the tables.

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DISCUSSION

DAVID F. SEIDERS*: Most of the literature on the GNMA futures market has been descriptive, and the "success" of the market too often has been measured solely in terms of the volume of activity. Professor Figlewski should be commended for addressing a question with important economic content. Indeed, the effect of GNMA futures markets on the stability of GNMA prices in the cash market can have significant consequences for the volume of residential mortgage and housing activity. Less stable spot prices for GNMAs, for example, could make these securities less attractive to investors, and higher average levels of yields on GNMAs—as well as on federally underwritten residential mortgages—could result.

The Figlewski paper concludes that the GNMA futures market has had a destabilizing effect on spot prices for GNMAs, even though competitive conditions prevail and price manipulation is not a factor. This conclusion is striking since such an effect had not been identified in previous studies of various commodity markets or in a prior study of the GNMA market. Moreover, the recent Treasury/Federal Reserve study of Treasury futures markets did not view destabilization of spot prices for Treasury securities to be a serious concern, at least under competitive market conditions.

Two arguments for destabilization appear to be intertwined in this paper, and it is useful to scrutinize them separately. One of the arguments, commonly offered in the context of competitive markets, runs as follows: futures markets encourage speculation by reducing the costs involved; speculators drive futures prices to levels out of line from market fundamentals; the swings in futures prices are

* Board of Governors, Federal Reserve System.

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