Journal of Real Estate Finance and Economics, 1: 5-22 (1988)

Valuing Mortgage Loan Servicing

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Key words: Mortgage, Servicing

Abstract

We develop an intertemporal model for valuing mortgage loan servicing contracts. The model includes a stochastic short-term interest rate and realized inflation rate which jointly determine the current mortgage coupon rate, the mortgagor's prepayment decision, the servicer's future net cash flows, and the rate at which to discount these future cash flows. Several potential uses of the model for institutions that service mortgages and trade servicing portfolios are illustrated by the application of the model to servicing fixed-rate mortgages and adjustable-rate mortgages. The model also is applicable to regulatory issues and to the servicing of other types of loans.

Mortgage loan servicing is a vital part of many financial institutions' operations, as well as a significant component of their portfolio of available investment opportunities.[1] For every loan originated and subsequently sold in the secondary market, a servicer agrees to collect the periodic payments from the borrower and pass the payments through to the holder of the loan. In return, the servicer retains a portion of each payment as a servicing fee. Mortgage loan originators may retain the servicing contract or sell the servicing contract separate from the loan. Mortgage servicers interested in enlarging their servicing portfolio often acquire servicing contracts from mortgage loan originators. In the secondary market for mortgage servicing both the buyer and the seller must value portfolios of servicing contracts.

As the secondary mortgage market has grown, and as the diversity of mortgage contracts has increased, analyzing and evaluating portfolios of loan servicing contracts has become an increasingly important and complex undertaking. In recent years, commercial banks, mutual savings banks, savings and loan associations, credit unions, and many traditionally nonfinancial institutions have joined

The authors gratefully acknowledge support from the Federal Home Loan Bank Board, Washington, D.C., the Purdue Research Foundation, West Lafayette, Indiana, and the Richard D. Irwin Foundation, Homewood, Illinois.

mortgage companies in buying and selling mortgage loan servicing contracts.[2] The emergence of these new mortgage loan servicers means that the valuation of mortgage servicing contracts is an important task, not only for the managers of these institutions, but also for the regulators charged with monitoring their activities.

In this paper we present a new framework for valuing mortgage loan servicing contracts for servicing both fixed rate mortgages (FRMs) and adjustable rate mortgages (ARMs). The paper is a synthesis and extension of two areas of the finance literature: the literature that discusses the value of mortgage loan servicing,[3] and the literature that develops intertemporal pricing models for default-free bonds and mortgage-related securities.[4] We treat mortgage servicing contracts as an investment opportunity whose value in an intertemporal world is determined by the interaction between future changes in the economy and optimal responses to those changes. Thus the paper represents a contribution to the literature which seeks to apply intertemporal pricing models to the problem of project evaluation. In that respect, our approach is similar in spirit to recent work by Brennan and Schwartz (1985b), Myers and Majd (1983), and Williams (1985). Each of these papers evaluates investment projects, explicitly taking into account the effect that changes in the economic environment can have on the investor's optimal decisions over the life of the project.

The loan servicing valuation model developed in this paper is a two state variable continuous-time model. The model includes a stochastic short-term interest rate which influences the current mortgage coupon rate, the mortgagor's prepayment decision, and the appropriate rate at which to discount the future servicing cash flows. The model also includes a stochastic aggregate price level (realized inflation rate) which determines, in part, the current mortgage coupon rate, the appropriate discount rate, and the magnitude of the servicer's nominal costs over the life of the servicing contract. Thus, in contrast to a static discounted-cash-flow technique, the appropriate discount rate and the mortgagor's prepayment decision are both determined as endogenous equilibrium conditions which are conditioned on the mortgage loan characteristics and the dynamics of the real interest rate and inflation rate.

The paper proceeds as follows. Section 1 contains a brief discussion of the structure of mortgage loan servicing contracts and how the values of the contracts are affected by an uncertain future interest rate and inflation rate. Section 2 presents the mortgage loan servicing valuation model, and section 3 illustrates several potential uses of the model with some simplified numerical examples. Section 4 discusses the relevance of the loan servicing valuation model to several issues that are closely related to the mortgage servicing industry. Concluding remarks appear in section 5.

1. Characteristics of mortgage loan servicing contracts

The value of a servicing contract primarily depends on three factors: the net cash flows of the contract, the rate at which to discount the cash flows, and the length of time the contract will be in effect. As discussed below, all three of these factors depend on uncertain future interest rates and inflation rates.

The income from servicing mortgage loans derives from a number of sources: the servicing fee paid by the mortgage loan investor, the interest earned on the float of the principal and interest (P&I) payments and the tax and insurance payments held in escrow, late charges which accrue to the servicer, transfer fees, assumption fees, and various other miscellaneous fees. The costs involved in servicing a loan include the costs of collecting the payments, submitting the payments and requisite reports to the loan holder, investigating and collecting delinquent payments, and in general protecting the lender's interests.

The most important source of servicing income is the servicing fee that the servicer receives from the investor who holds the mortgage loan.[5] The servicing fee is specified as a fixed percentage of the declining balance of the mortgage loan. However, if the aggregate price level is subject to inflation, the nominal costs incurred by the servicer are likely to vary over the life of the contract. As goods and services become more (or less) expensive, the servicer's expenditures on loan servicing also will fluctuate. The uncertain nominal servicing cost, combined with a fixed nominal servicing income, causes the net servicing cash flow to be a direct function of the inflation rate. Because the balance of the loan is declining over time, and since the inflation rate is likely to be positive over time, the net cash flow to the loan servicer is likely to decline over time and to potentially become negative.

The loan servicer also receives potentially valuable income in the form of interest income on the float from principal and interest payments and tax and insurance escrows. The value to the servicer of the float from P&I payment and escrows depends on the opportunity costs of funds, which in turn depend on the current short-term interest rate. Although these indirect sources of income are small relative to the mortgage, they may have significant value relative to the other servicing income, especially when current interest rates are high.

The discount rate, at which the future servicing cash flows are valued, is also a function of the real interest rate and inflation rate. *Ceteris paribus,* the appropriate discount rate is higher (lower) for periods in which the real interest rate or inflation rate is expected to be higher (lower).

Perhaps the most important factor that influences the value of a mortgage loan servicing contract is the possibility that the loan will be terminated prior to its original maturity date. When the loan is terminated, the servicing cash flows also are truncated, and the servicing contract has no remaining value. Loans are terminated at the mortgagor's option in two ways: the mortgagor may choose to default on the loan, or the mortgagor may choose to prepay the loan. The possibility of each type of loan termination affects the *ex ante* value of a loan servicing contract.

One reason that mortgagors prepay loans is to allow refinancing at a lower interest rate. The probability that a loan will be prepaid is related to the loan's coupon rate, the prepayment penalty (if any), and the refinancing alternatives available to the mortgagor. However, the refinancing alternatives are described by the uncertain real interest rate and inflation rate over the life of the loan. Equivalently, the refinancing alternatives depend on the current market value of the existing loan. If the market value is high, then refinancing with a similar maturity loan is attractive.[6]

2. The loan servicing valuation model

The value of a mortgage loan servicing contract depends directly on the stochastic interest rate and inflation rate, as well as on the current market value of the mortgage loan. However, the market value of the loan itself is also determined in equilibrium as a function of the stochastic interest rate and inflation rate. Thus the value of the mortgage loan and the value of the mortgage loan servicing contract are jointly determined in equilibrium.[7]

The argument that the value of the loan and the value of the servicing contract are jointly determined leads to a parity relationship that must hold in equilibrium. The parity relationship states that the value of the loan servicing contract is equal to the difference between the value of the loan when it is held by a servicer and the value of the loan when it is held by a non-servicing investor. That is, the value of the servicing contract is given by the spread between the value of the two forms of mortgage loan. The equilibrium values of the two slightly different versions of the loan completely describe the equilibrium value of the contract to service the loan.

The parity relationship can be expressed as

$$V(L_s) = V(L_i) + V(SC). \tag{1}$$

In (1), $V(L_s)$ gives the value of the mortgage loan when it is held by the servicer, $V(L_i)$ gives the value of the mortgage loan when it is held by the non-servicing investor, and $V(SC)$ gives the value of the mortgage loan servicing contract. In equilibrium, the value of the loan servicing contract must be such that the parity relationship holds. If the relationship does not hold, and $V(L_s) > V(L_i) + V(SC)$, then a loan servicer could buy the underlying loan and realize an immediate gain. Or, conversely, if $V(L_s) < (V(L_i) + V(SC)$, then a servicer who also holds the underlying loan cold sell the loan and realize an immediate gain. Thus, in the absence of such gains, the value of the loan servicing contract must satisfy the parity relationship in (1).[8]

The parity relationship is the basis for the loan servicing valuation model developed in this paper. To implement the loan servicing valuation model with the parity relationship, it is necessary to value the servicer-held and the investor-held mortgage loans.

2.1. The mortgage loan price equations

The model used to value both the investor-held and servicer-held loans is an application of Cox, Ingersoll, and Ross' (1985b) nominal bond pricing model. Their framework is applicable to our problem because it allows for both a stochastic nominal interest rate and stochastic realized inflation rate. The stochastic nominal rate is useful because it allows us to endogenize the mortgagor's prepayment decision. The stochastic realized inflation rate allows the pricing of nominal contracts which have some cash flows that are subject to inflation, and other cash flows that are not influenced by inflation. An unattractive feature of the model is that all the variation in the short-term riskless nominal interest rate is due to variation in the short-term real rate.

The primary assumptions used to derive the bond-pricing equation are a perfect and competitive capital market in which transactions take place costlessly and only at equilibrium prices, and a continuously operating capital market which allows the market participants to rebalance their investment portfolios at any point in time.[9]

The instantaneous riskless real interest rate, $r(t)$, is stochastic with dynamics

$$dr(t) = K_r r(t)dt + \sigma_r\sqrt{r(t)}\,dy \tag{2}$$

In (2), K_r determines the real interest rate drift, σ_r determines the variance of the real interest rate, and dy is a univariate Weiner process.[10]

There exists a variable, p, called the aggregate price level. The percentage change in p is called the inflation rate.[11] The dynamics of p are given by

$$dp/p = \pi dt + \sigma_p dz. \tag{3}$$

The expected rate of change in p is $E(dp/p) = \pi dt$, so that π is the expected rate of inflation. The variance of the inflation rate is determined by σ_p.[12]

Denote $N(r,p,t,T)$ as the equilibrium real price of a nominal bond at real interest rate r, aggregate price level p, time t, and with maturity date T. Define the time to maturity as $\tau = T - t$. The price of any default-free nominal bond or contract (i.e., mortgage loans) is described by the partial differential equation

$$\tfrac{1}{2}\sigma_r^2 rN_{rr} + \tfrac{1}{2}\sigma_p^2 p^2 N_{pp} + (K_r - \lambda)rN_r + \pi pN_p - N_\tau - rN + \Delta(r,p,t,T) = 0 \tag{4}$$

In (4), subscripts denote partial derivatives, and $\Delta(r,p,t,T)$ is the real value of the contract's payoff at real interest rate r, aggregate price level p, and time t. The parameter λ specifies the equilibrium bond return premium over the riskless rate. The assumptions and pricing equation above imply that the equilibrium nominal risk-free interest rate is given by $R(t) = r(t) + \pi - \sigma_p^2$. The nominal interest rate equals the sum of the real interest rate and the expected inflation rate minus the variance of the realized inflation rate.

The above pricing equation is a special case of the Cox, Ingersoll, and Ross (1985b) nominal term structure model. The nominal yield to maturity on a pure discount nominal bond is given by the $Y(r,t,T)$ such that $\exp[-(T - t)Y(r,t,T)] = N(r,p,t,T)p(t)$, where, in this case, $N(r,p,t,T)$ gives the price of a pure discount nominal bond with maturity date T. Solving for the yield to maturity gives

$$Y(r,t,T) = [B(\tau)/\tau]r(t) + \pi - \sigma_p^2 \tag{5}$$

where $B(\tau) = 2(e^{\gamma\tau} - 1)/[(\gamma + K_r + \lambda)(e^{\gamma\tau} - 1) + 2\gamma]$ and $\gamma = [(K_r + \lambda)^2 + 2\sigma_r^2]^{1/2}$. The nominal yield to maturity is increasing in K_r and π and decreasing in σ_r. Thus, in this model, the same economic factors that affect the value of servicing contracts and mortgages also determine the nominal term structure.

The solution to (4), subject to the appropriate $\Delta(r,p,t,T)$ and boundary and maturity conditions, gives the equilibrium value of a wide variety of default-free servicer-held and investor-held mortgage loans.[13] The term $\Delta(r,p,t,T)$ includes all loan payments for principal and interest, as well as the servicing cost, in the case of the servicer-held loan, and the servicing income, in the case of an investor-held loan. The exact form of $\Delta(r,p,t,T)$ is the subject of the following section.

Mortgage contracts that give the borrower the option to prepay the loan prior to maturity are modeled as in Brennan and Schwartz (1977) and Dunn and McConnell (1981a, 1981b). A mortgagor will choose to refinance if the future real payments can be reduced at no cost. A mortgagor can reduce future payments if the existing mortgage can be prepaid and refinanced at a lower coupon rate. With no transaction costs or refinancing costs, if mortgagors act optimally the loan will be prepaid whenever the market value of the remaining payments is greater than the remaining principal.[14] Denote the nominal amount of the unpaid principal at τ to maturity as $F(\tau)$. In effect, the borrower can, at any time, pay the mortgage holder $F(\tau)/p(t)$ for a mortgage with market value $N(r,p,t,T)$. Since individuals who trade mortgages are assumed to know that mortgagors will act optimally when making prepayment decisions, the market value of a prepayable mortgage loan will never be greater than the remaining principal. The refinancing prepayment constraint is written

$$N(r,p,t,T) \leqslant F(\tau)/p(t), \forall t. \tag{6}$$

Solving (4) subject to this prepayment constraint yields the equilibrium value of a prepayable mortgage loan.[15]

The servicing contract valuation model expresses the value of a mortgage servicing contract as the difference between the market value of the mortgage when it is held by the mortgage servicer and the market value of the mortgage when it is held by a non-servicing investor. The value of both types of mortgages is described by (4), subject to the prepayment constraint (6) and the maturity and boundary conditions. The difference in the solutions to (4) using the appropriate $\Delta(r,p,t,T)$ for investor-held and servicer-held mortgages gives the value of servicing the mortgage loan.

2.2. The cash flows

Both a non-servicing investor and a servicing investor receive the monthly principal and interest (P&I) payments from the mortgagor. However, the servicing investor not only incurs the costs associated with servicing the loan but also receives the benefits of the float on tax and insurance escrows received from the mortgagor. The non-servicing investor pays a fee to the servicer and potentially loses some float on the P&I payments due to the lag between the time the servicer receives the payments from the mortgagor and the time the investor receives the payments from the servicer. The P&I payment float lost by the non-servicing investor is a gain to the servicer, and thus may add to the value of loan servicing.

2.2.1. ARM and FRM principal and interest payments. The typical ARM has coupon payments that are non-Markovian; i.e., at any time during the life of the loan, the size of the payment depends on past outcomes of the nominal interest rate. Path dependence occurs because, on every coupon rate revision date, the size of the next period's coupon payment is calculated based on the unamortized principal on the revision date. However, on any given date the remaining principal is determined by the entire path of loan payments and coupon rates that has occurred up to that time. If the past coupon rates have been high on average, then the unamortized principal on any date will be greater than if the past coupon rates have been low on average. For ARMs, the historic path of coupon rates depends on the past path of the riskless interest rate. Thus the size of the monthly coupon payment on an ARM is non-Markovian.[16]

To use the pricing model in (4), the $\Delta(r,p,t,T)$ must be a function only of current states variables.[17] To implement the loan-pricing model, we specify the coupon payment to be non-Markovian. It is assumed that the coupon payment is given by

$$c(r,t) = B\rho(t)[1 - \exp(-\rho_a(T - t))]/\{[1 - \exp(-\rho(t)(T - t))] \times [1 - \exp(-\rho_a(T - t_0))]\} \quad (7)$$

where:

$$\begin{aligned} \rho(t) &= \rho_{\max} && \text{if} \quad R(t) + \varepsilon > \rho_{\max}, \\ &= R(t) + \varepsilon && \text{if} \quad \rho_{\min} \leqslant R(t) + \varepsilon \leqslant \rho_{\max}, \\ &= \rho_{\min} && \text{if} \quad R(t) + \varepsilon < \rho_{\min}. \end{aligned}$$

The payment structure in (7) is similar to the payments on a coupon-rate-cap ARM with the current coupon rate, $\rho(t)$, pegged at ε above the current nominal interest rate, and subject to a minimum rate of $\rho_{\min}$ and a maximum rate of $\rho_{\max}$. Equation (7) implies that there is no per-period adjustment caps or floors. In (7), B denotes the loan amount, t_0 the origination date, T the maturity date, and $\rho_a = (\rho_{\min} + \rho_{\max})/2$ is defined as the "average" coupon rate over the life of the loan.[18] Here, the assumed principal outstanding at τ years to maturity is given by $F(\tau) = B(1 - \exp(-\rho_a\tau))/[1 - \exp(-\rho_a(T - t_0)]$.

The assumed coupon payment in (7) can be used to price a variety of mortgage loans by choosing the appropriate ρ_{min}, ρ_{max}, ε, $T - t_0$, and B. For example, with $\rho_{min} = \rho_{max}$, (7) gives the coupon payment of a FRM. By varying $\rho_{max} - \rho_{min}$, ARMs with differing levels of coupon rate adjustability can be examined.

2.2.2. Servicer-held mortgage cash flows. Aside from receiving the P&I payments, the servicing investor incurs the servicing cost as well as gains the value of the float on the tax and insurance escrows. We assume that the real servicing costs are fixed over the life of the loan and are independent of loan size. That is, over the life of the loan, changes in the nominal servicing costs will depend on the inflation rate. We use *SVCST* to denote the annual amount of real servicing costs per mortgage loan.

Float on tax and insurance escrows is valuable to the servicer if it can be used as compensating balances on commercial bank loans. Thus the value of the float will be partially determined by the servicer's opportunity cost of funds. If the opportunity cost of funds is a function of the short-term riskless interest rate, the value of the float can be incorporated into the servicing valuation model. The value of the float will be correlated with the loan size to the extent that loan size is related to the mortgaged property's value, and will also depend on the amount of time the float is usable by the servicer. Let m be the fraction of the year that the float is usable by the servicer, and let the size of the annual tax and insurance payments be given by x times the original amount of the loan, B. Assume that the servicer's opportunity cost of funds is the short-term riskless interest rate. Then the rate of interest income of the tax and insurance escrows at nominal interest rate $R(t)$ is given by $R(t)Bx/m$.

In sum, the rate of the net cash flow accruing to the servicer-held mortgage loan is given by $\Delta_s(r,p,t,T) = \{[c(r,t) + (r(t) + \pi - \sigma_p^2)Bx/m]/p(t)\} - SVCST$.

2.2.3. Investor-held mortgage cash flows. Aside from receiving the P&I payments, the non-servicing investor pays a servicing fee to the loan servicer and, potentially, loses some float on the P&I payments. The servicing fee is almost always expressed as a percent of the remaining loan principal for each month the loan is outstanding. Denote *SF* as the annual rate of the servicing fee expressed as a percentage of the remaining loan principal; then the rate of the real servicing fee at time t is given by $SF \cdot F(\tau)/(p(t) \cdot 12)$.

The value of the P&I payment float that is given up by the non-servicing investor is a function of the size of the payments, the investor's opportunity cost of funds, and the amount of time the servicer can use the float. If the opportunity cost of funds is given by the short-term riskless interest rate, then the value of this float can be incorporated into the servicing valuation model. Denote n as the fraction of the year that the servicer can use the P&I payment float (that is, the fraction of the year that each P&I payment is not available to the non-servicing investor); then the interest yield given up by the non-servicing investor is given by $R(t)c(r,t)/np(t)$.

In sum, the rate of the net cash flow accruing to the investor-held mortgage loan is given by $\Delta_i(r,p,t,T) = [c(r,t) - SF \cdot F(\tau)/12 - (r(t) + \pi - \sigma_p^2)c(r,t)/n]/p(t)$.

The value of the servicer-held and investor-held mortgage loans is described in (4), along with either $\Delta_s(r,p,t,T)$ or $\Delta_i(r,p,t,T)$, and the maturity and boundary conditions. Implementing the mortgage servicing valuation model involves specifying the parameters that describe the economic environment and the loan contract, determining the current coupon rate for that environment and loan contract, and calculating the dollar value of the difference between the solutions to the partial differential equations describing the value of the servicer-held and investor-held mortgage loans.[19,20]

According to the valuation model, the mortgagor's optimal prepayment decision, including its effect on the value of servicing the mortgage, is determined endogenously as a function of the real interest-rate dynamics, the aggregate price level dynamics, and the terms of the loan contract. The net servicing cash flows are partially determined by the aggregate price level. The nominal servicing *fee* is certain; however, the nominal servicing *cost* is subject to inflation over the life of the servicing contract. Furthermore, the rate at which servicing cash flows are discounted is determined by the interest rate and aggregate price level dynamics and by the risk of the cash flows of the underlying mortgage. Finally, given the economic environment and the loan contract, the model determines and uses the current coupon rate. Thus the servicing valuation model incorporates the interaction of the key determinants of the value of mortgage loan servicing.

3. Some uses of the valuation model

The servicing valuation model is potentially useful in several ways, two of which are illustrated in this section. First, the model is potentially useful for examining the way in which changes in the economic environment affect the value of servicing newly-originated mortgage loans. Second, the model is potentially useful for examining the value of servicing mortgages (or mortgage pools) which were originated at a coupon rate different than the current coupon rate.

The following numerical examples are strictly illustrative for the following three reasons. First, the parameters used as inputs into the solutions presented are not empirical estimates. Ideally, empirical estimates should be used, but that is beyond the scope of the present paper. Second, the values obtained from the model are relatively sensitive to the parameter inputs used, and we have not performed an exhaustive study of the generality of the numerical solutions presented. Finally, for simplicity we have ignored both mortgage defaults and indirect servicing income (P&I payment and escrow float) in the numerical solution procedure.[21]

In the examples, the underlying mortgage loans considered all have a \$50,000 original balance and are of three types.[22] The first type is a standard 30-year fixed-rate mortgage loan (denoted FRM). Recall that a FRM is simply an ARM with ρ_{min}

$= \rho_{max}$. The second is a 30-year adjustable-rate mortgage loan (denoted ARM_L) that allows a low degree of adjustability. The maximum lifetime coupon rate adjustment is 400 basis points up or down from the original coupon rate ($\rho_{max} - \rho_{max} = .08$). The third is a 30-year adjustable-rate mortgage loan (denoted ARM_H) that allows a high degree of adjustability. The maximum lifetime coupon rate adjustment is 800 basis points up or down from the original coupon rate ($\rho_{max} - \rho_{min} = .16$).

3.1. Changes in the economic environment

Changes in the economic environment affect the term structure, the current mortgage coupon rate, and the value of servicing newly-originated mortgages. In the framework used in this paper, the economic environment is largely determined by the drift in the real rate (K_r), the variability of the real rate (σ_r), and the expected inflation rate (π). Thus, changes in these parameters will be evidenced by changes in the term structure, by changes in coupon rates of newly-originated prepayable mortgages, and by changes in the value of servicing these prepayable mortgages. The model developed in this paper is potentially useful for analyzing the effect of these economic factors on the value of mortgage servicing. Figure 1 contains some illustrations of the effect of changes in the above parameters on the value of mortgage servicing for the specific economic environment assumed.

Comparing the first two bars for each type of loan shows that in this simple case, an increase in the interest rate drift, K_r, may have a positive or negative effect on the observed values of loan servicing, depending on the adjustability of the mortgage coupon rate. An upward drifting interest rate has two offsetting effects on the value of servicing mortgage loans: a negative effect due to an increased discount rate, and a positive effect due to a decreased probability of mortgage refinancing prepayments. The two effects combined cause a 4% increase in FRM servicing value, an 11% decrease in ARM_L servicing value, and a 20% decrease in ARM_H servicing value. The increase in the value of servicing the FRM occurs because an upward drift in the interest rate reduces the probability of loan prepayment, thus increasing the value of servicing the loan. For the FRM, the increase in value caused by the reduced prepayment probability is larger than the decrease in value caused by an increased discount rate. For ARMs, the increase in servicing value caused by a decline in the probability of loan prepayment is more than offset by the decrease in servicing value caused by an upward drifting discount rate.

Figure 1 also illustrates that changes in the variability or uncertainty of the real interest rate may have a significant effect on the value of servicing portfolios of FRMs and ARMs. An increase in σ_r has two opposing influences on servicing value. On the one hand, an increase in σ_r implies a decrease in the discount rate used to evaluate future cash flows. On the other hand, an increase in σ_r causes an increase in the probability that the mortgage will be prepaid to allow refinancing at a lower interest rate. The first effect causes an increase in servicing value, while the second effect causes a decrease in servicing value.

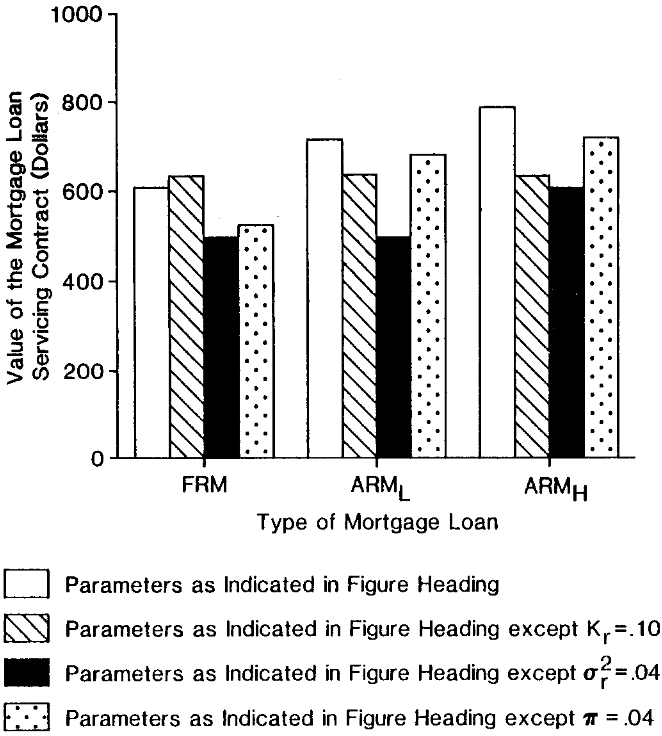

Fig. 1. Mortgage loan servicing contract dollar values for the servicing of three different $50,000 mortgage loans when $r(t) = .12, p(t) = 1.0, K_r = \pi = 0, \sigma_r^2 = .01, \sigma_p^2 = .02$, and $\lambda = -.10$ unless otherwise noted. The mortgage loans are assumed to be default-free, and the servicer is assumed to receive no float.

The first and third bars for each loan type in Figure 1 compare servicing values for prepayable loans for two different values of σ_r. According to the figure, an increase in interest rate variability causes a substantial decrease in the value of servicing both FRMs and ARMs. The decline occurs because an increase in σ_r causes an increase in the probability that the underlying mortgages will be prepaid for refinancing purposes. Since mortgage refinancing prematurely truncates the positive net cash flows, the *ex ante* value of servicing a portfolio of mortgages is diminished when it becomes more likely that refinancing will occur.

The fourth bar of each loan in Figure 1 contains servicing values for an economic environment identical to the first bar, except that the expected inflation rate is given by $\pi = .04$ instead of $\pi = 0$. The figure shows that an increase in the expected inflation rate causes a decline in the value of servicing portfolios of FRMs and ARMs. The decline in the value of servicing occurs because the servicing cash inflows are a fixed nominal amount, but the servicing cash outflows are subject to inflation. A rise in the aggregate price level increases nominal servicing costs, but leaves nominal servicing revenue unchanged.

3.2. Differing coupon rates

Servicing contracts for mortgage loans with coupon rates different from the current coupon rate are often traded. The coupon rate on the mortgage (or pool of mortgages) is relevant to the loan servicer because it affects the mortgagor's prepayment behavior in the future. For example, servicing a pool of mortgages which are trading at a discount (i.e., a low coupon pool) will likely have a value different from that of servicing a pool of otherwise identical newly-originated mortgages. The model proposed above is potentially useful in analyzing the relative value of servicing loans with differing coupon rates. Of course, the relative value of servicing the two pools not only depends on the spread between the coupon rates on the two pools, but also on the dynamics of the interest rate and inflation rate and on the type of mortgage contract.

Figure 2 presents values of servicing prepayable $50,000 FRMs, ARM_Ls, and ARM_Hs that have differing coupon rates. The first bar of each loan type shows values for servicing mortgages that are newly-originated at the current coupon rate. The second bar of each loan type presents values for servicing mortgages that are identical to those in the first bar, except that they have a coupon rate 200 basis points below the current coupon rate. The figure shows that, for the assumed economic environment, the value of servicing a portfolio of newly-originated mortgage loans is likely to be lower than the value of servicing a portfolio of mortgages with coupon rates lower than the current rate. Lowering the coupon rate 200 basis points causes a 37% increase in FRM servicing value, a 21% increase in ARM_L servicing value, and 16% increase in ARM_H servicing value. The differential effect between FRMs and ARMs occurs because lowering the coupon rate 200 basis points reduces the probability of a FRM refinancing prepayment relatively more than it reduces the probability of an ARM refinancing prepayment.

4. Further uses of the loan servicing valuation model

4.1. Mortgage servicing and the FSLIC

The Federal Savings and Loan Insurance Corporation (FSLIC) has a vested interest in the operations of the institutions involved in mortgage lending and servicing. A major task of the FSLIC is the determination of the solvency or equity position of the various financial institutions it oversees. The task is difficult because the capitalized value of the institutions' servicing portfolio does not appear on the balance sheet. Thus, the FSLIC must estimate the value of servicing portfolios when attempting to define a bankrupt financial institution.

The mortgage loan servicing valuation model developed in this paper is potentially useful to the FSLIC in evaluating the equity position of financial institutions. The model may be useful for determining how the value of a servicing portfolio is affected by changes in the economy. The model allows an analysis of

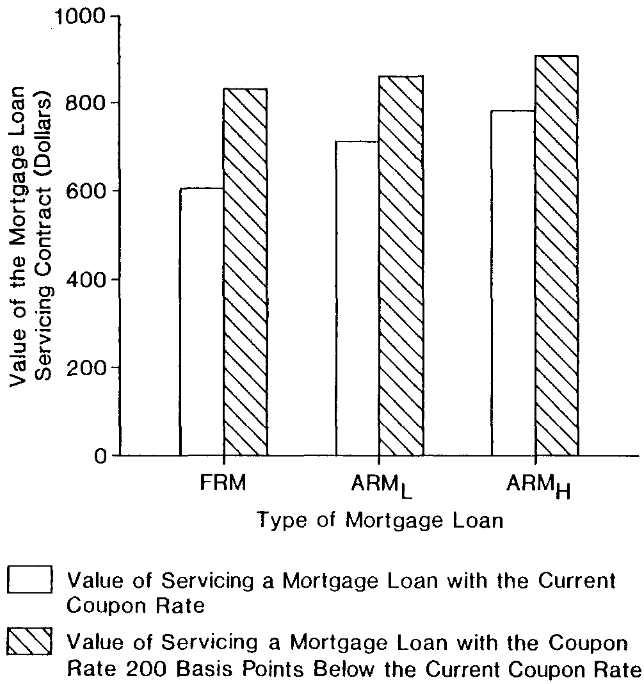

Fig. 2. Mortgage loan servicing contract dollar values when the mortgage coupon rate is below the current coupon rate. The values are for three different $50,000 mortgage loans when $r(t) = .12$, $p(t) = 1.0$, $K_r = \pi = 0$, $\sigma_r^2 = .01$, $\sigma_p^2 = .02$, and $\lambda = -.10$. The mortgage loans are assumed to be default-free, and the servicer is assumed to receive no float.

the interaction between the value of the servicing portfolio and the mortgage coupon rates, the mortgage contract, the inflation rate and the current interest rate.

Currently the FSLIC is analyzing the possibility of initiating deposit insurance premiums that are related to the risk of individual institutions. To the extent that a loan servicing portfolio is an important asset of an institution, an analysis of the institution's risk must include an analysis of the servicing portfolio. Such an analysis would be facilitated by a valuation model that explicitly incorporates the risks and uncertainties that affect the value of a servicing portfolio.

4.2. Mortgage servicing and the GNMA

The Government National Mortgage Association (GNMA) has an interest in mortgage servicers that is similar to that of the FSLIC. GNMA authorizes the issuance of mortgaged-backed pass-through securities that are backed by pools of insured mortgage loans. GNMA guarantees investors in GNMA securities that loan servicers will collect the loan payments and pas them through as long as the loan is outstanding. GNMA is responsible for the payments if the loan servicer fails to perform the appropriate servicing duties.

The underwriting risk to which GNMA is exposed is determined by the probability that loan servicers will default on their servicing obligations. However, servicer default is, in part, determined by the value of the servicing contract to the servicer. When servicing contracts have low or negative value, servicers are more likely to default. The valuation model developed here could be used to estimate the contingent liability assumed by GNMA, and, as a consequence, could be used to determine the appropriate premium that GNMA should assess for insuring mortgage loan servicing.

4.3. Applying the model to the servicing of other types of loans

The model for valuing mortgage loan servicing can be adapted to evaluate the servicing of various other types of loans in addition to the FRMs and ARMs discussed in this paper. Price-level-adjusted mortgages, graduated payment mortgages, commercial mortgage loans, consumer credit loans, automobile loans, and guaranteed student loans are all sold, with servicing released, in a secondary loan market. The model developed here could be used to evaluate the servicing of these loans, because the value of servicing the loans depends directly on the dynamics of the real interest rates and aggregate price level, and on the way in which the dynamics affect the borrowers' prepayment decisions.

Applying the servicing valuation model to a variety of loans can be accommodated by specifying the appropriate cash flows from the loan, and the applicable boundary conditions. The functional form of the cash flows, $\Delta(r,p,t,T)$, depends on the periodic loan payments, the expected prepayment and the default pattern, the servicing fee arrangement, and the costs incurred in servicing the loan. The maturity condition depends on the loan characteristics. If the loan is a non-amortizing coupon loan or a pure discount loan, the maturity condition is $N(r,p,T,T) = B$, where B is the face value of the loan. If the loan is prepayable, the applicable prepayment constraint becomes $N(r,p,t,T) \leqslant D(\tau)/p(t)$, where $D(\tau)$ is the prepayment price at τ years to loan maturity. Nonsystematic prepayments or defaults can be incorporated by using the Dunn and McConnell (1981b) approach and choosing the appropriate prepayment and default probabilities. Thus the servicing of a wide variety of loans can be evaluated with the same basic approach used in this paper to determine the value of servicing portfolios of FRMs and ARMs.

5. Conclusion

This paper proposes a new approach to evaluate mortgage loan servicing. The approach combines a simple parity relationship with standard intertemporal valuation techniques to develop a model that includes many of the relevant components of mortgage servicing value. The model allows the mortgagor's prepayment decision and the appropriate discount rate to be determined endogenously in equilibrium, as functions of the underlying state variables.

An application of the general model to the case of servicing fixed-rate mortgages and coupon-rate-cap adjustable-rate mortgages shows that the model is potentially useful for analyzing how the economic environment and the mortgage loan contracts interact to determine the value of servicing the loans. Illustrative numerical examples are provided for examining the way in which the value of mortgage loan servicing is affected by the interaction between the mortgage coupon rate, the type of mortgage loan contract, the prepayment incentives of the mortgagor, the real servicing cash flows, and the dynamics of the real interest rate and inflation rate. The standard discounted-cash-flow technique for evaluating projects (mortgage loan servicing) is not able to capture the dynamic multi-period interaction of the determinants of a project's value (such as the above factors in the mortgage loan servicing case). Intertemporal valuation models that allow for stochastic state variables and endogenously determined participant actions have the potential to provide better insight into project evaluation.

Notes

1. The dollar amount of mortgage loans being serviced by mortgage companies, savings and loans, commercial banks, mutual savings banks, life insurance companies, and Federal credit agencies exceeded $1.6 trillion in 1982 (from *Loans Closed and Servicing Volume for the Mortgage Banking Industry*, 1982, Tables 3 and 8).

2. A recent Wall Street Journal article (Dolan 1985) discusses the increasing activity in the secondary mortgage servicing market. The article states that Department of Housing and Urban Development statistics indicate that the servicing of about $24 billion in federally insured mortgages was transferred in the secondary market in the fiscal year ending September 30, 1985. According to the article Ford Motor Company, American Can, and Owens Illinois all entered the mortgage servicing industry in 1985, and General Motors Corporation is now "one of the nation's top mortgage servicers."

3. See, for example, Friedman (1977, 1978), McConnell (1976), and Miller and Meyer (1978).

4. See, for example, Brennan and Schwartz (1977, 1985a), Buser and Hendershott (1984), Buser, Hendershott, and Sanders (1985), Cox, Ingersoll, and Ross (1985b), Dunn and McConnell (1981a, 1981b), and Hendershott and Villani (1981).

5. Eighty-six percent of total servicing income in 1977 for 186 mortgage bankers was attributed to servicing fees received from the mortgage investors/holders (from *Income and Cost for Origination and Servicing of 1 to 4 Unit Residential Loans 1977*, Table 24). In the subsequent analysis, we ignore the income a loan servicer may receive from late charges, transfer fees, assumption fees, and other miscellaneous fees. Thus, the servicing contract values discussed in the remainder of the paper are net of these forms of income. Although these sources of income could, in principle, be incorporated into the model, it is unclear whether the analysis would be materially enhanced by the added structure.

6. Several papers that contain detailed theoretical discussions of the mortgagor's prepayment decision are Dunn and Spatt (1985, 1986), Hendershott, Hu, and Villani (1983), Siegel (1984), and Smith (1982). Empirical studies by Green and Shoven (1983) and Hendershott and Villani (1981) support the notion that the possibility of refinancing prepayments affects mortgage prices and coupon rates.

7. Loan servicing contracts are typically bundled together to form a portfolio of loan servicing contracts. This discussion could therefore also be in terms of servicing portfolios and the underlying mortgage pool. The model is applicable in either case.

8. The presence of a positive servicing contract value, $V(SC)$, does not imply that there exist positive economic rents. The value of a servicing contract represents a "quasi-rent," in that the originator/servicer incurs a net cost at the time of loan origination (the net cost is equal to the difference between the cost of origination and the origination fee and discount points received by the originator). McConnell (1977) provides a discussion and evidence that mortgage discount points at origination are part of the mechanism whereby mortgage originators/servicers earn a competitive rate of return on the origination and servicing of a variety of mortgage loan sizes.

9. For a full and detailed derivation of the nominal contract pricing model, see Cox, Ingersoll, and Ross' companion papers (1985a, 1985b).

10. Several choices are available for the form of the interest rate dynamics. Although the empirical support is still inconclusive, the most popular choice in the literature is a mean-reverting interest rate process. In this paper a simple non-mean-reverting process is used because it makes the numerical solution technique more tractable. Specifically, with the non-mean-reverting process the boundary condition for the partial differential equation in (4) below when $r = 0$ reduces to a simpler differential equation in p and τ.

11. The aggregate price level is assumed to have no real effects. However, if some contracts have real payoffs specified in terms of p (as do nominal bonds and mortgage loan servicing contracts), the real value of the contract will depend on p.

12. The univariate Weiner process dz is uncorrelated with the Weiner process dy, which determines the real interest rate outcomes. This implies that the price level is uncorrelated with the real interest rate.

13. The assumption that the mortgage loans being serviced are default free is made primarily for simplicity. Mortgage loan defaults which are uncorrelated with any market factors (and thus diversifiable) could be incorporated relatively easily by using an approach similar to Dunn and McConnell (1981b). With that approach the mortgage would be priced based on the expected default outcome at each date prior to maturity. We chose not to include this type of default because it would add structure to the model and little, if any, additional insight. Mortgage loan defaults which are systematic could be modelled with the use of an additional state variable describing house values and a corresponding default constraint, as described in Kau, Keenan, Muller, and Epperson (1986a). However, the additional state variable would make the model intractable.

14. Dunn and Spatt (1986) analyze the mortgagor's optimal prepayment behavior when the mortgagor must pay a fixed fee to refinance the mortgage. In Dunn and Spatt's refinancing cost framework it is not optimal to refinance in the manner assumed in this paper.

15. The following conditions also must be satisfied when solving (4) to determine the value of a mortgage loan. The maturity condition is expressed as $N(r,p,T,T) = 0$ for mortgages that are fully amortized at the scheduled maturity. With fixed or capped loan payments, as the real interest rate increases, the present value of the future payments declines. Under this boundary condition, $N(r,p,t,T)$ approaches zero as r approaches infinity. As the aggregate price level increases, the purchasing power of a fixed or capped nominal receipt declines. Under this boundary condition, $N(r,p,t,T)$ approaches zero as p approaches infinity.

16. There exists two general types of ARM contracts: those with payment caps, and those with coupon rate caps. In this paper we consider the more popular rate-cap ARMs.

17. Kau, Keenan, Muller, and Epperson (1986b) argue that a mortgage with path dependent payments can be modelled in a framework similar to the one used in this paper. However, their method requires the introduction of an additional state variable, which would make the analysis in this paper intractable.

18. The coupon payments given by (6) differ from actual rate-cap payments because of the assumed constant amortization rate, ρ_a. In actuality, the amortization rate changes over the life of the ARM. Buser, Hendershott, and Sanders (1985) bound the possible ARM coupon payment outcomes by assuming that the ARM is non-amortizing (ρ_a approaching infinity), and that the ARM is linearly amortized ($\rho_a = 0$). They show that the mortgage price is not very sensitive to the assumed ρ_a. The ρ_a used in this paper is always within these bounds, thus allowing for a reasonable analysis of the value of mortgage servicing.

19. The current coupon rate is defined as the coupon rate which, for a given economic environment and mortgage loan contract, allows the mortgage loan to sell at part when issued. Thus the current coupon rate is endogenous to the model.

20. Since no analytic solution is known to exist, the partial differential equations are solved numerically. The procedure is to find the boundary conditions when $r = p = 0$, $r = 0$, and $p = 0$, by numerically solving the resulting ordinary differential equation and partial differential equations. Once the boundary values are known, (4) can be numerically solved, subject to the boundary values and the conditions in footnote 15.

21. The accuracy of the numerical solution technique is difficult to assess in general. We did, however, perform a rudimentary test of the solution algorithm and software by comparing the computer-generated solution with the analytic solution for the simple case where the analytic solution is known. In the perfect certainty case ($K_r = \pi = \sigma_r = \sigma_p = \lambda = 0$), the analytic solution differed from the solution generated by the numerical technique by less than .013%. This gives some confidence that there are no errors in the numerical solution technique.

22. In all the following examples the servicing fee is assumed to be .0044/12 of the remaining loan principal in each month. In general, the size of the servicing fee depends on the specific servicing contract agreed upon. The fee usually ranges between .125% and .625% of the remaining principal per year (which is converted to a monthly fee by dividing by 12). The servicing cost per loan is assumed to be $40 at the time of the loan origination. Detailed aggregate data is available on mortgage bankers' servicing costs from the Mortgage Bankers Association of America (from *Income and Cost of Origination and Servicing of 1 to 4 Unit Residential Loans 1977*, Table 24). Data from 186 mortgage bankers in 1977 indicate that the total direct servicing cost for these mortgage bankers was $36.60 per loan, and the overhead allocation was an additional $14.00 per loan.

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