The Impact of Group Purchasing Organizations on Healthcare-Product Supply Chains

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Abstract. This paper examines the impact of group purchasing organizations (GPOs) on healthcare-product supply chains. The supply chain we examine consists of a profit-maximizing manufacturer with a quantity-discount schedule that is nonincreasing in quantity and ensures nondecreasing revenue, a profit-maximizing GPO, a competitive source selling at a fixed unit price, and n providers (e.g., hospitals) with fixed demands for a single product. Each provider seeks to minimize its total purchasing cost (i.e., the cost of the goods plus the provider's own fixed transaction cost). Buying through the GPO offers providers possible cost reductions, but may involve a membership fee. Selling through the GPO offers the manufacturer possibly higher volumes, but requires that the manufacturer pay the GPO a contract administration fee (CAF); i.e., a percentage of all revenue contracted through it. Using a game-theoretic model, we examine questions about this supply chain, including how the presence of a GPO affects the providers' total purchasing costs. We also address the controversy about whether Congress should amend the Social Security Act, which, under current law, permits CAFs. Among other things, we conclude that although CAFs affect the distribution of profits between manufacturers and GPOs, they do not affect the providers' total purchasing costs.

1 Introduction

Group purchasing organizations (GPOs) play a very important role in the supply chains for healthcare products. A survey by Burns and Lee (2008) reports that nearly 85% of U.S. hospitals route 50% or more of their commodity-item spending, and 80% route 50% or more of their pharmaceutical spending through GPOs. According to the Health Industry Group Purchasing Association (HIGPA),

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“nearly every hospital in the U.S. (approximately 96% to 98%) chooses to utilize GPO contracts for their purchasing functions.”

U.S. healthcare-product GPOs started in the late 1800s, and grew rapidly in the late 1970s and early 1980s, partly in response to competition from for-profit hospitals, and partly in response to pressure to reduce costs from third-party payers. GPOs evolved to become significant “players” in healthcare-product supply chains following a 1987 amendment to the Social Security Act, which permits GPOs to charge contract administration fees (CAFs) to manufacturers. CAFs are essentially commissions paid by manufacturers to GPOs on sales to the GPO’s members. Prior to the 1987 amendment, CAFs had been prohibited.

CAFs are controversial. They are criticized by manufacturers, who complain that they are forced to charge higher prices for all products, whether they are sold through a GPO or not, in order to recover the CAFs paid to GPOs. Sethi (2006) estimates that “... GPOs generate excess revenue in the range of $5-6B, which legitimately belongs to its members ...” Singer (2006), in reference to the so-called “safe harbor” provisions of the 1987 amendment, says “the elimination of the safe harbor (provisions) ... would generate large savings for the federal government.” Others assert that CAFs create a conflict of interest, i.e., that GPOs do not have an incentive to negotiate the lowest possible prices for their members because the CAF is based on that negotiated price.

The fundamental rationale for joining a GPO is that a provider will incur a lower total purchasing cost—that is, the cost of the given product plus the provider’s own fixed transaction cost, or *fixed contracting cost*—by buying through the GPO than by contracting for that same item directly with a manufacturer. GPOs assert that they are able to lower their provider-members price per unit by employing market intelligence and product expertise that no single member could afford, and by contracting for the group’s combined purchase quantity. GPOs are able to lower a provider’s fixed contracting cost by spreading its own, presumably higher, fixed contracting cost over its many members. Schneller and Smeltzer (2006, p. 218, Table 1.3) identify several components of a provider’s fixed contracting cost, among them determining product use and requirements, preparing bids or requests for proposal, and conducting product evaluation.

Schneller and Smeltzer (2006) report that a provider’s fixed contracting cost is $3,116 per contract if a provider contracts directly with a manufacturer, and $1,749 per contract if a provider contracts through a GPO: a difference of $1,367 in fixed contracting cost per contract. We refer to this difference as the *GPO's contracting efficiency*; i.e., the reduction of the provider's fixed contracting cost if it contracts through a GPO instead of contracting directly with a manufacturer. It should be noted that Schneller and Schmeltzer's estimate of GPO contracting efficiency ($1,367) is based on case studies conducted by Novation, a GPO, and cannot be independently confirmed. Nonetheless, no one in the industry questions the contracting efficiency of GPOs, not even GPO critics, such as Sethi (2006) and Singer (2006) cited above.

GPOs earn revenue from several sources: membership fees charged to provider-members, CAFs charged to manufacturers, administrative fees charged to distributors authorized to distribute products under the GPOs' contracts, and miscellaneous service fees. According to Burns and Lee (2008), GPO membership fees are "nonnegligible" for providers; e.g., $300,000 – $600,000 for a small hospital system anchored around a teaching hospital. However, CAFs are the primary source of GPO revenues (Burns 2002, p. 69).

In this paper, we assume that healthcare providers have four important characteristics, which we believe represent practice. First, each provider seeks to minimize its total purchasing cost (i.e., the cost of the goods purchased plus the provider's own fixed transaction cost), not merely the cost of the goods themselves. Second, we assume that each provider's demand, denoted as its *purchasing requirement*, is fixed. Third, providers who belong to GPOs are not required to purchase specific products through the GPO. Hence, providers are free to negotiate directly with manufacturers or other suppliers. Fourth, providers are highly diverse, particularly with respect to size, and hence, the size of their purchasing requirements. Regardless of these differences in size, there is evidently enough homogeneity in other respects to provide common ground for both small and large providers to belong to healthcare GPOs (Arnold 1997). Of course, our results apply to the extent that these assumptions hold. See §8 for comments on the impact of our paper's major assumptions and limitations on our results.

Given the significant, controversial role that GPOs play in healthcare-product supply chains, our research asks the following questions: (1) Do providers experience lower prices or lower total purchasing costs with a GPO in the supply chain? (2) Do CAFs mean higher prices paid by providers? (3) How does the presence of the GPO affect manufacturer profits? (4) What affects GPO profits? (5) How are supply-chain profits divided between the manufacturer and the GPO, and how is the division influenced by the "power" of the GPO? The answers to these questions have implications for government policy and, in practical terms, for the cost of healthcare.

In order to explore these issues, we analyze a game-theoretic model that includes a profit-maximizing GPO, a profit-maximizing manufacturer whom the GPO has already chosen to contract with, and n providers with various purchasing requirements. We also assume the presence of a competitive source that sells the product at a fixed exogenous price. The salient features of our model, which in combination are characteristics of healthcare GPOs, are: (1) the GPO's contracting efficiency; (2) CAFs that GPOs charge to manufacturers for GPO-contracted sales; and (3) GPO membership fees.

The sequence of events in our model is as follows. First, the manufacturer and the GPO negotiate the size of the CAF that the manufacturer will pay the GPO for on-contract purchases. We do not model this negotiation directly. Instead, the CAF is a parameter whose value represents the "power" of the manufacturer versus the GPO; e.g., the higher the CAF, the more powerful the GPO. Note that the "safe harbor" provisions of the 1987 amendment nominally limit CAFs to 3%; however, exceptions are permitted. Second, given the CAF, the providers' purchasing requirements, and the price from the competitive source, the manufacturer determines a quantity-discount schedule.

Third, the GPO determines what on-contract price to offer providers in order to maximize its profit. The GPO collects a CAF from the manufacturer for on-contract sales, and may charge a fixed membership fee to the providers who decide to buy through it. In order to represent the GPO as a profit maximizer, we have modeled it as buying from the manufacturer at one price and selling to its members at another price. In fact, GPOs neither buy nor sell products. Instead, they negotiate the on-contract prices that their members pay for a manufacturer's products. Hence, if the manufacturer agrees, the member's on-contract price can be set higher than the manufacturer's own price for that quantity, thereby generating a positive margin for the GPO.

Finally, each provider splits its requirement among the GPO, the manufacturer, and the competitive source, in order to minimize its total purchasing cost. We assume that the providers incur the same fixed contracting cost when buying from the manufacturer or from the competitive source but a lower fixed contracting cost when buying through the GPO because of the GPO's contracting efficiency.

Before continuing, a few comments about our model. First, modeling a supply chain with a single GPO is reasonable. Although Burns and Lee (2008) report that 41% of the providers surveyed belong to more than one GPO, they "... route most of their purchases through a single national alliance ... and utilize (another) only for specific contracts in limited supply areas." Second, GPO contracts are typically "rebid" every 3 to 5 years, depending on the GPO and the type of product. We do not model this bidding process. Instead, our model assumes that bidding has already taken place for a given item and that a single manufacturer has been chosen by the GPO to sell products to its members. We do not model what the GPO does with its profits. It should be noted that some GPOs are member-owned or partially member-owned. In such cases, the provider-members receive a share of the GPO's profits. We also do not account for the possibility that large providers may negotiate a portion of the GPOs' CAF. We will return to these last two points in §8. We also do not examine the question of GPO formation since, as already noted, virtually every provider already belongs to a GPO. Hence, the important questions do not involve GPO formation but the impact of a GPO on the providers' costs and the circumstances under which providers will avail themselves of GPO procurement services.

The remainder of our paper is organized as follows. First, we define the game in its most general form: with a manufacturer that offers a quantity-discount schedule that is nonincreasing in quantity and ensures nondecreasing revenue, and with n providers with heterogeneous fixed purchasing requirements. For simplicity, we have assumed that the manufacturer's production cost is zero. However, our results continue to hold, provided that the manufacturer's marginal profit is marginally decreasing in quantity, and the manufacturer's total profit is increasing in quantity (See §3). An analysis of the subgame-perfect Nash equilibrium strategies, specifically Lemmas 4.1 and 4.2, provides a key structural result. This result provides the basis of an algorithm for computing an equilibrium solution for any parameterization of the general case.

Following this analysis of the general case, we fully characterize the equilibrium of two special cases: the case of two heterogeneous providers, and the case of n identical providers, when the manufacturer's quantity-discount schedule is linear. The analysis of these special cases provides unambiguous answers to the questions posed above. Then, using the algorithm in §4, we compute equilibria using similar parameterizations, but in more general cases; specifically, n heterogeneous providers with a linear discount schedule, 2 heterogeneous providers with a nonlinear discount schedule, and n identical providers with a nonlinear discount schedule. We observe that the characteristics of these equilibria are similar to the cases for which we have analytical solutions.

Based on these observations, we *believe* these results on equilibria can be generalized (see §7).

2 Literature review

Healthcare GPOs have been discussed in the healthcare-management literature for years. Burns (2002) and Schneller and Smeltzer (2006) provide qualitative analyses of healthcare-GPO structure and function. More recently, Burns and Lee (2008), through a large-scale survey of hospital material managers, examined GPOs from the viewpoint of their members. They report: (1) 94% of survey respondents belong to a GPO, and (2) GPOs succeed in reducing health care costs by lowering product prices, particularly for commodity and pharmaceutical items.

Two U.S. Government Accountability Office (GAO) reports (2003, 2010) provide background information about GPO business practices and the “safe harbor” provision in the Social Security Law. The 2003 report describes processes that GPOs use to select manufacturers’ products, the CAFs that they charge to manufacturers, their use of contracting strategies to obtain favorable prices, etc. The 2010 report describes the types of services that GPOs provide to members, how they fund these services, etc. An earlier GAO pilot survey from 2002, one often cited by GPO critics, reported that “a hospital’s use of a GPO contract did not guarantee that the hospital saved money: GPO’s prices were not always lower and were often higher than prices paid by hospitals negotiating with vendors directly.” This is one of the controversies that our analysis addresses.

Our model, along with Hu and Schwarz (2011), provides the first theoretical analyses of healthcare GPOs. Hu and Schwarz (2011) examine some of the controversies about GPOs through the Hotelling model: a continuum of identical providers and two manufacturers. The providers decide whether to form a GPO when negotiating a price with the manufacturers. They show that forming a GPO increases competition between manufacturers, thus lowering prices for healthcare providers. They also demonstrate that the existence of lower off-contract prices is not, per se, evidence of anticompetitive behavior on the part of GPOs. Indeed, under certain circumstances, the presence of a GPO lowers off-contract prices. They also examine the consequences of eliminating the “safe harbor” provisions and conclude that it would not affect any party’s profits or costs.

Due to limitations of the Hotelling model used in Hu and Schwarz (2011), hospitals are treated as identical, each having the same purchasing requirement. In contrast, our model has a discrete number of providers who may have different requirements, thus allowing us to examine the impact of the differences in healthcare providers' purchasing requirements. In contrast to Hu and Schwarz (2011) where the GPO is formed by the providers, here the GPO is an independent entity that negotiates contracts for the providers by charging membership fees and CAFs, thereby possibly making a profit. Hence, to the best of our knowledge, our model captures important features that have not been examined in the current healthcare supply chain literature.

One strand of economics literature examines the impact on competition among manufacturers when buyers form a GPO to commit to purchasing exclusively from only one of the manufacturers. This strand of research does not address the features of healthcare supply chains that are identified in the introduction, such as CAFs and the price inelasticity of the buyers' demands, nor is the GPO independent from the buyers as it is in our models. However, like our model, the models in this literature have a GPO formed by the buyers, which can potentially aggregate the purchasing requirements of its members and commit to purchasing only from one manufacturer. This is equivalent to sole-sourcing, a practice of GPOs that is often criticized as anti-competitive. O'Brien and Shaffer (1997) show that buyers can obtain lower prices through both nonlinear pricing and sole sourcing, which intensify competition between the rival suppliers. Dana (2003) extends O'Brien and Shaffer (1997) by endogenizing the decisions of buyers to form groups. He shows that if the GPO commits to purchasing exclusively from one supplier, then the buyers obtain a lower price, one that is equal to the suppliers' marginal costs. Both papers show that exclusive-dealing or sole sourcing is a mechanism that empowers the GPO to negotiate a lower price for its members and therefore is not anti-competitive.

Marvel and Yang (2008) study a similar problem, assuming that: (1) the GPO's interests are aligned with the buyers and thus seeks to minimize the buyers' total purchasing costs; and (2) the sellers have the bargaining power, offering take-or-leave it nonlinear pricing tariffs to the GPO. Unlike Dana (2003), the GPO in their model cannot identify individual providers' utilities. They demonstrate that the competition-intensifying effect of the nonlinear tariff, not the GPO's bargaining power, lowers the GPO's purchasing price since the sellers have the bargaining power in their model.

There is a vast operations/supply-chain management literature on contracting—see Cachon (2003), for example—but very little involves GPOs or other contracting intermediaries. Wang et al. (2004) discuss channel performance when a manufacturer sells its goods through a retailer using consignment contracts with revenue sharing. Assuming a monopoly manufacturer who offers a linear quantity discount to competing retailers, Chen and Roma (2008) identify conditions under which a GPO will form. In all these papers, the retailers' demands are price-elastic, depending on the retail prices.

Another stream of research concerns the allocation of alliance benefits back to its members, the fairness of allocation, and the stability of the alliance through a cooperative game framework. In particular, Schotanus et al. (2008) and Nagarajan et al. (2008) study how a GPO can allocate cost savings among its members. The latter further discusses the stability of the GPO under different allocation rules.

Except for the 2002 GAO pilot study cited above, there have been no empirical studies of GPO pricing. The 2002 GAO study was criticized with respect to its scope (e.g., a small number of products) and methodology (e.g., failure to account for GPO contracting efficiency). According to a U.S. Senate Minority Staff Report (2010), "... in 2009, Senator (Charles) Grassley asked the GAO to examine 50 or more medical devices and supplies to evaluate the impact of GPO contracting on pricing. The GAO subsequently informed Committee staff that '... it was unable to establish a methodology that would address the concerns raised about its 2002 pilot study.'" Hence, there are no empirical studies besides the GAO's 2002 pilot, and unless the GAO changes its position, such an independent empirical study is not likely to be forthcoming.

3 Game-theoretic models

We consider two non-cooperative games, one that includes a GPO in the purchasing process, and one that does not. Both games have complete information; that is, every player knows the payoffs of all the other players.

3.1 With a GPO

In this non-cooperative game, there are $n + 2$ players: the manufacturer, the GPO, and the n providers. First, the manufacturer offers the same quantity-discount schedule to the GPO and the n providers. Then, the GPO determines the on-contract price for its provider-members. Finally, each provider $i \in \{1, \ldots, n\}$ determines how much of its requirement to buy (a) through the GPO, (b) directly from the manufacturer, or (c) from the competitive source. Tables 1 and 2 summarize the parameters and decision variables we use to describe the game.

We assume that the providers are indexed according to nondecreasing purchasing requirements;

Table 1: Summary of parameters

q_i	provider i's fixed purchasing requirement, for $i = 1, \dots, n$
$\hat{p}$	the competitive source's fixed unit price
$\hat{f}^{\mathrm{G}}$	GPO membership fee
$\tilde{f}^{\mathrm{G}}$	each provider's fixed contracting cost when purchasing through the GPO
f^{G}	$= \hat{f}^{\mathrm{G}} + \tilde{f}^{\mathrm{G}}$
f^{M}	each provider's fixed contracting cost when purchasing from the manufacturer or competitive source
λ	CAF ($0 \leq \lambda \leq 1$)

Table 2: Summary of decision variables

u_i	1 if provider i purchases through the GPO, and 0 otherwise
v_i	1 if provider i purchases from the manufacturer, and 0 otherwise
w_i	1 if provider i purchases from the competitive source, and 0 otherwise
x_i	quantity purchased by provider i through the GPO
y_i	quantity purchased by provider i from the manufacturer
z_i	quantity purchased by provider i from the competitive source
p^{G}	GPO's per-unit on-contract price
$p(\cdot)$	manufacturer's quantity-discount schedule: for a quantity q, the manufacturer offers a price of $p(q)$

that is, $q_1 \leq q_2 \leq \cdots \leq q_n$. In addition, we assume $f^{\mathrm{G}} \leq f^{\mathrm{M}}$: the providers' fixed contracting cost is lower through the GPO. We denote $\Delta f = f^{\mathrm{M}} - f^{\mathrm{G}} \geq 0$ the *GPO's contracting efficiency.*

We formally describe the game by defining the optimization problem for each player. For each $i = 1, \dots, n$, provider i's problem is to minimize the total cost of purchasing q_i (a) through the GPO, (b) directly from the manufacturer, and (c) from the competitive source:

$$\begin{aligned} \pi_i = \min \quad & \underbrace{\left(f^{\mathrm{G}} u_i + p^{\mathrm{G}} x_i\right)}_{\text{(a)}} + \underbrace{\left(f^{\mathrm{M}} v_i + p(y_i) y_i\right)}_{\text{(b)}} + \underbrace{\left(f^{\mathrm{M}} w_i + \hat{p} z_i\right)}_{\text{(c)}} \\ \text{s.t.} \quad & x_i + y_i + z_i = q_i, \;\; x_i \leq q_i u_i, \;\; y_i \leq q_i v_i, \;\; z_i \leq q_i w_i, \\ & u_i \in \{0,1\}, \;\; v_i \in \{0,1\}, \;\; w_i \in \{0,1\}, \;\; x_i \geq 0, \;\; y_i \geq 0, \;\; z_i \geq 0. \end{aligned} \tag{3.1}$$

The GPO's problem is to choose the unit on-contract price that maximizes its profit:

$$\pi_{\mathrm{G}} = \max \quad \underbrace{\hat{f}^{\mathrm{G}} \sum_{i=1}^{n} u_i}_{\text{(a)}} + \underbrace{p^{\mathrm{G}} \sum_{i=1}^{n} x_i}_{\text{(b)}} - \underbrace{(1-\lambda) p\left(\sum_{j=1}^{n} x_j\right) \sum_{i=1}^{n} x_i}_{\text{(c)}} \qquad \text{s.t.} \qquad p^{\mathrm{G}} \geq 0.$$

The GPO's revenue consists of (a) membership fees and (b) on-contract sales, and the cost (c) it incurs is its provider-members' combined purchasing cost from the manufacturer, discounted by the

CAF.

Finally, the manufacturer's problem is to choose a quantity-discount schedule that maximizes its revenue (i.e., profit):

$$\pi_{\mathrm{M}} = \max \quad \underbrace{(1-\lambda)p\Bigg(\sum_{j=1}^{n} x_j\Bigg)\sum_{i=1}^{n} x_i}_{\text{(a)}} + \underbrace{\sum_{i=1}^{n} p(y_i)y_i}_{\text{(b)}} \tag{3.2}$$

$$\text{s.t.} \quad p(q) \text{ is nonincreasing in } q, \;\; p(q)q \text{ is nondecreasing in } q.$$

The manufacturer's revenue is derived from sales: (a) through the GPO, discounted by the CAF, or (b) directly to the providers. We assume that the manufacturer's choice of quantity-discount schedule $p(q)$ is constrained so that it is nonincreasing in the quantity q, and the associated revenue $p(q)q$ is nondecreasing in q. In addition, we assume that when a provider can purchase its requirement at the same total purchasing cost from each of its three options, its preference is first, to buy through the GPO, second, to purchase directly from the manufacturer, and third, to purchase from the competitive source.

3.2 Without a GPO

The non-cooperative game without a GPO is very similar. There are $n+1$ players: the manufacturer and the n providers. The competitive source remains exogenous with the same unit price. The sequence of events are similar to those in §3.1, except that the GPO is absent.

For each $i = 1, \ldots, n$, provider i's problem is to minimize its total purchasing cost:

$$\pi_i = \min \quad \big(f^{\mathrm{M}} v_i + p(y_i)y_i\big) + \big(f^{\mathrm{M}} w_i + \hat{p} z_i\big)$$

$$\text{s.t.} \quad y_i + z_i = q_i, \;\; y_i \le q_i v_i, \;\; z_i \le q_i w_i, \;\; v_i \in \{0,1\}, \;\; w_i \in \{0,1\}, \;\; y_i \ge 0, \;\; z_i \ge 0.$$

The manufacturer's problem is still to choose a quantity-discount schedule that maximizes its revenue:

$$\pi_{\mathrm{M}} = \max \quad \sum_{i=1}^{n} p(y_i)y_i \quad \text{s.t.} \quad p(q) \text{ is nonincreasing in } q, \;\; p(q)q \text{ is nondecreasing in } q.$$

Note that both games described in this section assume that the manufacturer's production cost is zero, but can be easily extended to include this cost. By reinterpreting $p(q)$ as the manufacturer's unit *profit* function in the quantity q, including the manufacturer's production cost does not affect our analysis, as long as the manufacturer's marginal profit is nonincreasing, and its total profit is nondecreasing in quantity. This holds given our assumptions on the quantity-discount schedule, for example, when the manufacturer's production cost is linear in quantity.

In the next section, we use backward induction to reveal the structure of equilibrium strategies for these games.

4 The structure of equilibrium strategies

4.1 With a GPO

The following lemma states that in an SPNE, each provider, in choosing the option with the lowest total purchasing cost, purchases its *entire* requirement either (a) from the GPO, (b) directly from the manufacturer, or (c) from the competitive source. Let s_i represent provider i's sourcing strategy, and let "GPO," "mfr," and "comp" represent these respective sourcing options.

Lemma 4.1. *Let $p(\cdot)$ and p^{G} be any given strategies for the manufacturer and the GPO, respectively. Then, for $i = 1, \ldots, n$:*

a. if $p^{\mathrm{G}} \le p(q_i) + \frac{\Delta f}{q_i}$ and $p^{\mathrm{G}} \le \hat{p} + \frac{\Delta f}{q_i}$, then $u_i = 1$, $x_i = q_i$, $v_i = 0$, $y_i = 0$, $w_i = 0$, $z_i = 0$ (i.e. $s_i =$ GPO) is an optimal strategy for provider i;

b. if $p(q_i) + \frac{\Delta f}{q_i} < p^{\mathrm{G}}$ and $p(q_i) \le \hat{p}$, then $u_i = 0$, $x_i = 0$, $v_i = 1$, $y_i = w_i$, $w_i = 0$, $z_i = 0$ (i.e. $s_i =$ mfr) is an optimal strategy for provider i;

c. if $\hat{p} + \frac{\Delta f}{q_i} < p^{\mathrm{G}}$ and $\hat{p} < p(q_i)$, then $u_i = 0$, $x_i = 0$, $v_i = 0$, $y_i = 0$, $w_i = 1$, $z_i = q_i$ (i.e. $s_i =$ comp) is an optimal strategy for provider i.

We define the *break-even price* p_k^{B} of provider $k \in \{1, \ldots, n\}$ as the price at which provider k is indifferent between purchasing through the GPO and the less costly of its other two direct purchasing options; that is,

$$p_k^{\mathrm{B}} = \min\{p(q_k), \hat{p}\} + \frac{\Delta f}{q_k} \qquad \text{for } k = 1, \ldots, n. \tag{4.1}$$

Let π_k^{B} be the GPO's profit at break-even price p_k^{B}: that is,

$$\pi_k^{\mathrm{B}} = k\hat{f}^{\mathrm{G}} + p_k^{\mathrm{B}} \sum_{i=1}^{k} q_i - (1-\lambda) p\left(\sum_{j=1}^{k} q_j\right) \sum_{i=1}^{k} q_i \qquad \text{for } k = 1, \ldots, n. \tag{4.2}$$

For notational convenience, we define $p_0^{\mathrm{B}} = +\infty$ and $\pi_0^{\mathrm{B}} = 0$: this price and corresponding profit captures the possibility that the GPO can set its price sufficiently high so that all providers find it cheaper to purchase directly from the manufacturer or the competitive source.

Using Lemma 4.1, we can characterize the optimal strategies of the providers and the GPO as a function of the manufacturer's quantity-discount schedule. Note that since p is nonincreasing in q and $q_1 \leq \cdots \leq q_n$, there exists ℓ' such that $p(q_i) > \hat{p}$ for all $i = 1, \ldots, \ell'$, and $p(q_i) \leq \hat{p}$ for all $i = \ell' + 1, \ldots, n$.

Lemma 4.2. *Let $p(\cdot)$ be any given strategy for the manufacturer. Let ℓ' be such that $p(q_i) > \hat{p}$ for all $i = 1, \ldots, \ell'$, and $p(q_i) \leq \hat{p}$ for all $i = \ell' + 1, \ldots, n$.*

a. The strategy $\mathrm{p}^{\mathrm{G}} = p^{\mathrm{B}}_{k'}$ is optimal for the GPO, where $k' = \arg\max_{k=0,1,\ldots,n} \pi^B_k$, with associated payoff

$$\pi^B_{k'} = \max\left\{0, \max_{k=1,\ldots,n}\left\{k\hat{f}^G + p^{\mathrm{B}}_k \sum_{i=1}^{k} q_i - (1-\lambda)p\left(\sum_{j=1}^{k} q_j\right)\sum_{i=1}^{k} q_i\right\}\right\}.$$

b. Suppose $\mathrm{p}^{\mathrm{G}} = p^{\mathrm{B}}_{k'}$ is an optimal strategy for the GPO, for some $k' \in \{0, \ldots, n\}$. Then the provider i's optimal strategy is

$$s_i = \mathit{GPO} \text{ for } i = 1, \ldots, k'; \quad \mathit{comp} \text{ for } i = k'+1, \ldots, \ell'; \quad \mathit{mfr} \text{ for } i = \ell'+1, \ldots, n.$$

Lemma 4.2 states that in equilibrium, it is optimal for the GPO to set its unit on-contract price to a break-even price. In addition, if it is optimal for a provider to purchase through the GPO (manufacturer) in equilibrium, then it is optimal for all providers with smaller (larger) purchasing requirements to also purchase through the GPO (manufacturer). Intuitively, because each provider's demand is fixed and known, the manufacturer and the GPO offer prices to extract as much profit as possible from providers, whose tradeoff is between a lower unit price (either from the competitive source $\hat{p}$, from the GPO, or directly from the manufacturer) and the fixed saving from contract efficiency. As a result, if a provider purchases through the GPO, all the smaller providers will do so as well. The manufacturer's equilibrium strategy is as follows.

Lemma 4.3. *Define $k'(p)$ so that the break-even price $p^{\mathrm{B}}_{k'(p)}$ is an optimal strategy for the GPO—i.e., as defined in Lemma 4.2a—when the manufacturer's quantity-discount schedule is p. In addition, for any quantity-discount schedule p, define $\ell'(p)$ so that $p(q_i) > \hat{p}$ for all $i = 1, \ldots, \ell'(p)$, and $p(q_i) \leq \hat{p}$ for all $i = \ell'(p) + 1, \ldots, n$. Then, the manufacturer's optimal strategy is*

$$\arg \max_{p(q):\,\frac{\partial p}{\partial q} \leq 0,\, \frac{\partial p(q) q}{\partial q} \geq 0} \left\{(1-\lambda)p\left(\sum_{j=1}^{k'(p)} q_j\right)\sum_{j=1}^{k'(p)} q_j + \sum_{j=\ell'(p)+1}^{n} p(q_j)q_j\right\} \tag{4.3}$$

Using Lemmas 4.1, 4.2 and 4.3 with an exhaustive search through all the manufacturer's feasible quantity-discount schedules, we can compute an SPNE of this game. Suppose $\mathcal{P}$ is a finite set of feasible quantity-discount schedules that contains the manufacturer's optimal strategy (4.3). Depending on the nature of the quantity-discount schedule, this can be achieved by discretizing the space of quantity-discount schedules sufficiently fine. How this can be done for particular quantity-discount schedules is discussed in §7. The procedure in Figure 1 computes an SPNE $(s', p^{\mathrm{G}\prime}, p')$.

$\pi_{\mathrm{M}} \leftarrow +\infty$
for all $p \in \mathcal{P}$ **do**
  Find ℓ' such that $p(q_i) > \hat{p}$ for $i = 1, \ldots, \ell'$, and $p(q_i) \leq \hat{p}$ for $i = \ell' + 1, \ldots, n$.
  Find $k' = \arg\max_{k=0,1,\ldots,n} \pi_k^{\mathrm{B}}$, where π_k^{B} is defined in (4.2).
  Compute $\pi_{\mathrm{M}}(p) = (1-\lambda)p(\sum_{j=1}^{k'} q_j)\sum_{j=1}^{k'} q_j + \sum_{j=\ell'+1}^{n} p(q_j)q_j$.
  if $\pi_{\mathrm{M}}(p) > \pi_{\mathrm{M}}$ **then**
    $\pi_{\mathrm{M}} \leftarrow \pi_{\mathrm{M}}(p)$
    $s'_i \leftarrow$ GPO for $i = 1, \ldots, k'$
    $s'_i \leftarrow$ comp for $i = k'+1, \ldots, \ell'$
    $s'_i \leftarrow$ mfr for $i = \ell'+1, \ldots, n$
    $p^{\mathrm{G}\prime} \leftarrow p_{k'}^{B}$ as defined in (4.1)
    $p' \leftarrow p$
  end if
end for

Figure 1: Procedure for computing an SPNE of the non-cooperative game with a GPO.

4.2 Without a GPO

As in the game with a GPO, we can show that in equilibrium, each provider purchases its *entire* requirement either (a) from the manufacturer, or (b) from the competitive source, choosing the least costly. In addition, if it is optimal for a provider to purchase through the manufacturer in equilibrium, then it is optimal for all providers with larger purchasing requirements to also purchase through the manufacturer.

Lemma 4.4. *Let $p(\cdot)$ be any given strategy for the manufacturer. Let ℓ' be such that $p(q_i) > \hat{p}$ for all $i = 1, \ldots, \ell'$, and $p(q_i) \leq \hat{p}$ for all $i = \ell'+1, \ldots, n$. Then:*

a. $v_i = 0$, $y_i = 0$, $w_i = 1$, $z_i = q_i$ (i.e. $s_i =$ comp) is an optimal strategy for providers $i = 1, \ldots, \ell'$;

b. $v_i = 1$, $y_i = q_i$, $w_i = 0$, $z_i = 0$ (i.e. $s_i =$ mfr) is an optimal strategy for providers $i = \ell'+1, \ldots, n$.

Based on Lemma 4.4, we obtain the following characterization of the manufacturer's equilibrium strategy.

Lemma 4.5. *For any quantity-discount schedule p, define $\ell'(p)$ so that $p(q_i) > \hat{p}$ for all $i = 1, \ldots, \ell'(p)$, and $p(q_i) \le \hat{p}$ for all $i = \ell'(p)+1, \ldots, n$. Then, the manufacturer's optimal strategy is*

$$\arg \max_{p(q):\, \frac{\partial p}{\partial q} \le 0,\, \frac{\partial p(q) q}{\partial q} \ge 0} \left\{ \sum_{j=\ell'(p)+1}^{n} p(q_j) q_j \right\}. \tag{4.4}$$

In a similar fashion to the game with a GPO, Lemmas 4.4 and 4.5 together with an exhaustive search of the manufacturer's feasible quantity-discount schedules imply a procedure for computing a subgame perfect Nash equilibrium of this game, similar to the one in Figure 1.

In the next two sections, we use these structural insights to fully characterize the equilibrium behavior for two special cases, both with a linear quantity-discount schedule: two providers with different purchasing requirements and n providers with identical purchasing requirements. We will then return our attention to more general cases and identify insights from these special cases that appear to apply more generally.

5 The case of two heterogeneous providers and linear quantity discount

In this section, we focus on the special case with two heterogeneous providers. We assume that the manufacturer offers a linear quantity-discount schedule $p(\cdot)$ of the form

$$p(q) = p^* - \gamma q, \tag{5.1}$$

where p^* is the manufacturer's unit base price, exogenously given. We assume $p^* > \hat{p}$, since otherwise, no provider would purchase from the competitive source. The manufacturer's decision variable is γ, the discount rate.

We are not claiming that such schedules are, in fact, linear in practice. However, linear quantity-discount schedules are widely used in the literature (e.g., Nagarajan et al. 2008; Chen and Roma 2008). In our model, linear discounts allow us to analytically characterize equilibrium behavior, and as we shall see in §7, these characterizations appear to apply in the case of nonlinear quantity-discount schedules as well.

In the game with a GPO, the manufacturer's optimization problem (3.2) can be rewritten as

$$\pi^{\mathrm{M}} = \max \quad (1-\lambda)\left(p^* - \gamma \sum_{j=1}^{n} x_j\right) \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} (p^* - \gamma y_i) y_i \quad \text{s.t.} \quad 0 \le \gamma \le \gamma^{\max}, \tag{5.2}$$

where $\gamma^{\max} = p^*/(2\sum_{i=1}^{n} q_i)$. The manufacturer's choice of discount rate γ is constrained between 0 and $\gamma^{\max}$ so that its revenue $p(q)q$ as a function of the quantity q is nondecreasing on $[0, \sum_{i=1}^{n} q_i]$. We also assume that $\hat{p} \ge p^*(1 - q_1/(2\sum_{i=1}^{n} q_i))$, or equivalently, that for each provider $i = 1, \ldots, n$,

$p^* - \gamma q_i \le \hat{p}$ for some $\gamma \in [0, \gamma^{\max}]$; that is, we assume that it is feasible for the manufacturer to set its discount rate so that its unit price for each provider is less than the competitive source's unit price. The manufacturer's optimization problem for the game without a GPO can be written in a similar way. In addition, we assume $n = 2$, and $q_1 < q_2$. We call provider 1 the "small provider" and provider 2 the "large provider."

5.1 Equilibrium behavior with a GPO

We characterize the subgame perfect Nash equilibria (SPNE) of the game with a GPO by backwards induction. First, we define some parameters. Let

$$\gamma^{(1)} = \frac{(q_1+q_2)((1-\lambda)p^* - \hat{p}) - 2\hat{f}^{\mathrm{G}} - (1+\frac{q_1}{q_2})\Delta f}{(1-\lambda)(q_1+q_2)^2}, \quad \gamma^{(2)} = \frac{q_2((1-\lambda)p^* - \hat{p}) - \hat{f}^{\mathrm{G}} - \frac{q_1}{q_2}\Delta f}{(1-\lambda)q_2(2q_1+q_2)},$$

$$\Delta f^{(1)} = \frac{q_1 q_2^2 (q_1+q_2)(p^* - \hat{p}) - q_2(q_2^2 - q_1^2 + 2q_1q_2)\hat{f}^{\mathrm{G}}}{q_2^3 - q_1^3 + 2q_1q_2^2} - \frac{q_1 q_2^2 (q_1+q_2)p^*}{q_2^3 - q_1^3 + 2q_1q_2^2}\lambda,$$

$$\Delta f^{(2)} = \frac{q_2^2(p^* - \hat{p}) - q_2\hat{f}^{\mathrm{G}}}{q_1} - \frac{q_2^2 p^*}{q_1}\lambda.$$

The different SPNE of this game can be categorized according to the level of the GPO's contracting efficiency. We say that the GPO's contracting efficiency is "low" if $\Delta f \in [0, \Delta f^{(1)}]$, "moderate" if $(\Delta f^{(1)}, \Delta f^{(2)}]$, and "high" if $\Delta f \in (\Delta f^{(2)}, +\infty)$. (In Lemma A.1, we show that $\max\{\Delta f^{(1)}, 0\} \le \max\{\Delta f^{(2)}, 0\}$, so this characterization makes sense. In the same lemma, we also show that the relative size of $\gamma^{(1)}$, $\gamma^{(2)}$, and $\gamma^{(3)}$ depend on the magnitude of Δf.)

Building upon the results in Section 4, we characterize the SPNE of the game.

Theorem 5.1 (Characterization of SPNE, two heterogeneous providers, with a GPO)**.** *Given different levels of contracting efficiency, the following strategy profiles are SPNE, with their associated payoffs:*

	strategies				*payoffs*			
contracting efficiency	s_1	s_2	p^{G}	γ	π_1	π_2	π_{G}	π_{M}
"low" ($\Delta f \le \Delta f^{(1)}$)	*GPO*	*GPO*	$\hat{p} + \Delta f / q_2$	$\gamma^{(1)}$	$\pi_1^{(1)}$	$\pi_2^{(1)}$	*0*	$\pi_{\mathrm{M}}^{(1)}$
"moderate" ($\Delta f^{(1)} < \Delta f \le \Delta f^{(2)}$)	*GPO*	*GPO*	$\hat{p} + \Delta f / q_2$	$\gamma^{(2)}$	$\pi_1^{(1)}$	$\pi_2^{(1)}$	$\pi_{\mathrm{G}}^{(1)}$	$\pi_{\mathrm{M}}^{(2)}$
"high" ($\Delta f > \Delta f^{(2)}$)	*GPO*	*GPO*	$\hat{p} + \Delta f / q_2$	*0*	$\pi_1^{(1)}$	$\pi_2^{(1)}$	$\pi_{\mathrm{G}}^{(2)}$	$\pi_{\mathrm{M}}^{(3)}$

where

$$\pi_1^{(1)} = q_1\hat{p} + f^G + \frac{q_1}{q_2}\Delta f, \quad \pi_2^{(1)} = q_2\hat{p} + f^M,$$

$$\pi_{\rm G}^{(1)} = \frac{q_2^2 - q_1^2 + 2q_1q_2}{q_2(2q_1+q_2)}\hat{f}^G + \frac{q_1(q_1+q_2)}{2q_1+q_2}\big(\hat{p} - (1-\lambda)p^*\big) + \frac{(q_1+q_2)(q_2^2 - q_1^2 + q_1q_2)}{q_2^2(2q_1+q_2)}\Delta f,$$

$$\pi_{\rm G}^{(2)} = 2\hat{f}^G + (q_1+q_2)\big(\hat{p} - (1-\lambda)p^*\big) + \Big(1 + \frac{q_1}{q_2}\Big)\Delta f, \quad \pi_{\rm M}^{(1)} = (q_1+q_2)\Big(\hat{p} + \frac{\Delta f}{q_2}\Big) + 2\hat{f}^G,$$

$$\pi_{\rm M}^{(2)} = \frac{q_1(q_1+q_2)}{2q_1+q_2}(1-\lambda)p^* + \frac{(q_1+q_2)^2}{2q_1+q_2}\Big(\hat{p} + \frac{\hat{f}^G}{q_2} + \frac{\Delta f}{q_2}\frac{q_1}{q_2}\Big), \quad \pi_{\rm M}^{(3)} = (1-\lambda)(q_1+q_2)p^*.$$

Note that both providers always purchase from the GPO in equilibrium. This result generalizes to $n > 2$ providers with identical purchasing requirements (§6). However, this result does not apply in general. The price $p^{\rm G}$ that the GPO charges is the breakeven price for the large provider. The small provider benefits from the magnitude of the large provider's purchasing requirement: the total purchasing cost π_1 of the small provider decreases as the large provider's purchasing requirement q_2 increases. However, the large provider's total purchasing cost π_2 does not depend on the smaller provider's purchasing requirement q_1.

Also we see that the GPO's profit $\pi_{\rm G}$ is strictly positive when the contracting efficiency is moderate or high. When its contracting efficiency is low, the manufacturer collects all the payments from providers, and the GPO just breaks even (i.e., $\pi_{\rm M} = \pi_1 + \pi_2$ and $\pi_{\rm G} = 0$). When the contracting efficiency is moderate or high, the GPO is no longer a profitless intermediary, and the payments from the providers are split between the manufacturer and the GPO (i.e., $\pi_{\rm M} + \pi_{\rm G} = \pi_1 + \pi_2$ and $\pi_{\rm G} \geq 0$).

The GPO membership fee affects the players in different ways. For all levels of contracting efficiency, we observe the following for a given value of the total fixed contracting cost $f^{\rm G}$. As the GPO membership fee $\hat{f}^{\rm G}$ increases, the GPO's profit $\pi_{\rm G}$ and the manufacturer's profit $\pi_{\rm M}$ stay the same or increase. However, the providers' total purchasing costs π_1 and π_2 are not affected: the providers' total fixed contracting cost $f^{\rm G}$ remains the same, and a change in the membership fee only changes how much of $f^{\rm G}$ gets transferred to the GPO.

Interestingly, the manufacturer's effective unit price for the small provider $p^* - \gamma q_1$ and the large provider $p^* - \gamma q_2$ is *greater* than the competitive source's unit price $\hat{p}$. In particular, the proof of Theorem 5.1 indicates that the manufacturer's discount rate γ at equilibrium does not exceed $(p^* - \hat{p})/q_2$ and $(p^* - \hat{p})/q_1$. Despite this, the manufacturer still "gets the business" of the providers because of the GPO's contracting efficiency and aggregating abilities. Also, note that when the GPO's contracting efficiency is "high", the manufacturer's optimal strategy is to set a zero discount rate. In this regime, the GPO's contracting efficiency is so large that both providers will purchase through the GPO, regardless of the manufacturer's quantity-discount schedule, and so the manufacturer optimizes by not offering a discount at all.

Next, we more closely examine the behavior of the equilibria described in Theorem 5.1 with respect to the contracting administration fee λ and the contracting efficiency Δf. To facilitate the analysis, we define the following regions in $(\lambda, \Delta f)$-space:

$$\Xi^{\mathrm{L}} = \{(\lambda, \Delta f) : 0 \leq \Delta f \leq \Delta f^{(1)},\ 0 \leq \lambda \leq 1\}, \quad \Xi^{\mathrm{M}} = \{(\lambda, \Delta f) : \Delta f^{(1)} < \Delta f \leq \Delta f^{(2)},\ 0 \leq \lambda \leq 1\},$$
$$\Xi^{\mathrm{H}} = \{(\lambda, \Delta f) : \Delta f > \Delta f^{(2)},\ 0 \leq \lambda \leq 1\}.$$

Figure 2 illustrates the characterization of SPNE described in Theorem 5.1 in $(\lambda, \Delta f)$-space. The providers' costs, the GPO's profit, and the manufacturer's profit at equilibrium as functions of λ and Δf are:

$$\pi_1(\lambda, \Delta f) = \pi_1^{(1)} \quad \text{for all } (\lambda, \Delta f) \in \Xi^{\mathrm{L}} \cup \Xi^{\mathrm{M}} \cup \Xi^{\mathrm{H}},$$
$$\pi_2(\lambda, \Delta f) = \pi_2^{(1)} \quad \text{for all } (\lambda, \Delta f) \in \Xi^{\mathrm{L}} \cup \Xi^{\mathrm{M}} \cup \Xi^{\mathrm{H}},$$
$$\pi_{\mathrm{G}}(\lambda, \Delta f) = \begin{cases} 0 & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{L}}, \\ \pi_{\mathrm{G}}^{(1)} & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{M}}, \\ \pi_{\mathrm{G}}^{(2)} & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{H}}; \end{cases} \qquad \pi_{\mathrm{M}}(\lambda, \Delta f) = \begin{cases} \pi_{\mathrm{M}}^{(1)} & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{L}}, \\ \pi_{\mathrm{M}}^{(2)} & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{M}}, \\ \pi_{\mathrm{M}}^{(3)} & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{H}}. \end{cases}$$

In addition, in order to examine how the manufacturer and the GPO share the revenue coming from the providers, we define the *profit share of the GPO* as

$$\rho^{\mathrm{G}} = \frac{\pi_{\mathrm{G}}}{\pi_{\mathrm{M}} + \pi_{\mathrm{G}}}.$$

In the following corollary, we examine how these quantities behave as functions of the contracting efficiency, Δf, and the CAF, λ. Recall that the providers' total fixed contracting cost f^{G} consists of $\hat{f}^{\mathrm{G}}$, the GPO membership fee, and $\tilde{f}^{\mathrm{G}}$, the providers' fixed contracting cost when purchasing through the GPO. With f^{M} and $\hat{f}^{\mathrm{G}}$ fixed, an increase in the contracting efficiency $\Delta f = f^{\mathrm{M}} - f^{\mathrm{G}} = f^{\mathrm{M}} - \hat{f}^{\mathrm{G}} - \tilde{f}^{\mathrm{G}}$ corresponds to a decrease in $\tilde{f}^{\mathrm{G}}$.

Corollary 5.2. *Suppose f^M and $\hat{f}^G$ are fixed. Then:*

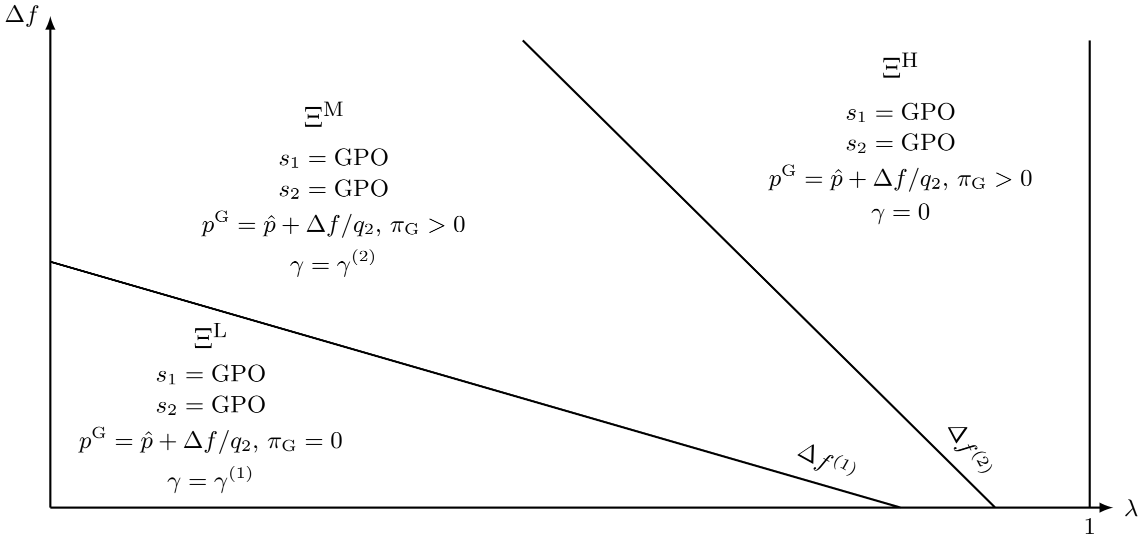

Figure 2: Characterization of the SPNE described in Theorem 5.1 in $(\lambda, \Delta f)$-space.

	$\pi_1(\lambda, \Delta f)$	$\pi_2(\lambda, \Delta f)$	$\pi_{\mathrm{G}}(\lambda, \Delta f)$	$\pi_{\mathrm{M}}(\lambda, \Delta f)$	ρ^G
as a fn. of Δf	*nonincreasing, linear*	*constant*	*nondecreasing, piecewise linear, convex*	*nondecreasing, piecewise linear, concave*	*nondecreasing*
as a fn. of λ	*constant*	*constant*	*nondecreasing, piecewise linear, convex*	*nonincreasing, piecewise linear, concave*	*nondecreasing*

First, Corollary 5.2 states that as the contracting efficiency increases, the small provider's total purchasing cost decreases, while the large provider's total purchasing cost stays the same. In addition, as the contracting efficiency increases, the profit of the GPO and the manufacturer increases. Therefore, an increase in contracting efficiency benefits all channel members. We expect this type of relationship for the GPO's profit, since the GPO "charges" for the contracting efficiency in its unit on-contract price, as shown in Theorem 5.1. Interestingly, this "charge" for contracting efficiency trickles up to the manufacturer as well. This phenomenon is evidently attributable to the fact that the manufacturer anticipates the GPO's response when determining its quantity-discount schedule, and as a result, is able to "capture" some of the GPO's contracting efficiency.

Next, consider the behavior of the equilibrium payoffs as the CAF varies. According to Corollary 5.2, neither provider's total purchasing cost is affected by the CAF; the CAF only affects the profits of the GPO and the manufacturer. As the CAF increases, the GPO's profit increases, while the manufacturer's profit decreases. However, their total profits remain unchanged because the providers' total costs are invariant to the CAFs. The higher the CAF, the more profitable the

GPO. Finally, as both the contracting efficiency and the CAF increase, the GPO captures a larger fraction of the revenue collected from the providers.

The top row of plots in Figure 5 (page 28) illustrates the behaviors described in Corollary 5.2 in $(\lambda, \Delta f)$-space. Note that lighter areas indicate lower values and darker areas, higher values. The diagonal lines in each plot are provided as a guide for comparison with Figure 2. Note that in the first two plots from the left, neither provider's total purchasing cost is affected by the CAF (i.e., the plots do not change shading along any horizontal line). The large provider's total purchasing cost is also unaffected by Δf (i.e., the plot does not change shading along any vertical line). However, the small provider's total purchasing cost is nonincreasing in Δf (i.e., the shading in the plot lightens going up any vertical line). GPO and manufacturer profit are displayed in the third and fourth plots of the same row: note that the GPO's profit is nondecreasing and the manufacturer's profit is nonincreasing in the CAF. Both the GPO and the manufacturer's profit is nondecreasing in Δf. In the fifth plot, the GPO's fraction of profit is nondecreasing in both λ and Δf. As already noted, in the sixth plot, both providers always purchase through the GPO in this scenario.

For more general cases (e.g., a nonlinear manufacturer quantity-discount schedule) we are unable to provide analytic results, but computational experiments indicate that provider total purchasing cost and GPO/manufacturer profit behave in a similar manner. See §7.

5.2 Equilibrium behavior without a GPO

Define $\hat{p}^{(1)} = \frac{q_2(q_2-q_1)}{q_1^2+q_2(q_2-q_1)}p^*$. The different SPNE of this game can be described in terms of the level of competition between the manufacturer and the competitive source. We say there is "high" competition if $\hat{p} \in [0, \hat{p}^{(1)}]$, and "low" competition if $\hat{p} \in (\hat{p}^{(1)}, p^*]$. For the game without a GPO, we have the following characterization of SPNE.

Theorem 5.3 (Characterization of SPNE, two heterogeneous providers, without a GPO)**.** *Given different levels of competition, the following strategy profiles are SPNE, with their associated payoffs:*

	strategies			*payoffs*		
competition	s_1	s_2	γ	π_1	π_2	π_{M}
"high" *($0 \le \hat{p} \le \hat{p}^{(1)}$)*	*comp*	*mfr*	$(p^* - \hat{p})/q_2$	$\pi_1^{(2)}$	$\pi_2^{(2)}$	$\pi_{\mathrm{M}}^{(4)}$
"low" *($\hat{p}^{(1)} < \hat{p} \le p^*$)*	*mfr*	*mfr*	$(p^* - \hat{p})/q_1$	$\pi_1^{(2)}$	$\pi_2^{(3)}$	$\pi_{\mathrm{M}}^{(5)}$

where

$$\pi_1^{(2)} = q_1\hat{p} + f^M, \quad \pi_2^{(2)} = q_2\hat{p} + f^M, \quad \pi_2^{(3)} = q_2\Big(\frac{q_2}{q_1}\hat{p} - \Big(\frac{q_2}{q_1} - 1\Big)p^*\Big) + f^M,$$

$$\pi_{\mathrm{M}}^{(4)} = q_2\hat{p}, \quad \pi_{\mathrm{M}}^{(5)} = q_1\hat{p} + q_2\big(\frac{q_2}{q_1}\hat{p} - (\frac{q_2}{q_1} - 1)p^*\big).$$

The equilibria described in Theorem 5.3 are driven by the usual trade-off between price and volume. When competition is low, the competitive source's unit price is relatively high. So in this scenario, the manufacturer can easily provide a unit price to both providers that is lower than the competitive source's, by setting a relatively low discount rate. On the other hand, when competition is higher, the competitive source's unit price is relatively low. So, in order to compete with the competitive source for both providers, the manufacturer must set a relatively high discount rate. As a result of the trade-off between price and volume, the manufacturer may find it more profitable to attract only the large provider.

5.3 The effect of a GPO's presence

The presence of a GPO affects the total purchasing cost of the providers in different ways. These differences are partly driven by the mechanisms that the manufacturer and the GPO use to price the product. Lemma 4.2 tells us that when it is optimal for the large provider to purchase through the GPO, it is optimal for the smaller provider as well. The opposite holds for purchasing from the manufacturer.

Comparing the equilibrium total purchasing costs of the providers with and without the presence of the GPO, we obtain the following corollary.

Corollary 5.4. (a) $\pi_1^{(1)} < \pi_1^{(2)}$; (b) $\pi_2^{(1)} = \pi_2^{(2)} > \pi_2^{(3)}$.

We see that the small provider benefits from the presence of the GPO: its total purchasing cost in the presence of a GPO is always strictly less than its total purchasing cost in the absence of a GPO. However, the large provider benefits from the absence of a GPO: its total purchasing cost in the absence of a GPO is no greater than that in the presence of a GPO. Moreover, when there is "low" competition, its total purchasing cost in the absence of a GPO is strictly less. This occurs since in the absence of a GPO, the large provider benefits when the manufacturer sets a higher discount rate in order to attract the small provider.

As discussed above, the providers face a *higher* unit price in the presence of a GPO. In the absence of a GPO, the effective unit price to the providers is $\hat{p}$, while in the presence of a GPO, the effective unit price to the providers is $\hat{p} + \Delta f/q_i$. However, this difference is offset by the lower contracting costs in the presence of a GPO. This result is consistent with the findings of a pilot study described in §2: that GPO prices were not always lower but often higher than prices paid by providers that negotiated directly with vendors. These GAO findings are used as criticisms of GPOs. However, as shown in our model, this result is consistent with providers seeking the lowest total purchasing cost but not necessarily the lowest unit cost.

In the absence of a GPO, the manufacturer may not be able to attract the business of both providers, while in the presence of a GPO, the manufacturer gets the business of both providers through the GPO. This occurs when competition is sufficiently high; that is, when the base unit price p^* and the competitive source's unit price $\hat{p}$ are so far apart that the manufacturer is unable to set a sufficiently high discount rate to compete with the competitive source.

6 The case of n identical providers and linear quantity discount

We now focus on the case with n identical providers, i.e, $q_1 = \cdots = q_n = q$ and a linear quantity-discount schedule. By symmetry, all n providers have the same equilibrium strategy and associated payoff. We denote this strategy simply by s_i and the associated payoff π_i.

First, we consider the equilibrium behavior in the game with a GPO. Define

$$\gamma^{(3)} = \frac{q(p^* - \hat{p}) - \hat{f}^{\mathrm{G}} - \Delta f}{(1-\lambda)nq^2} - \frac{p^*}{(1-\lambda)nq}\lambda, \qquad \Delta f^{(3)} = q(p^* - \hat{p}) - \hat{f}^{\mathrm{G}} - \lambda q p^*.$$

In this case, we say that the GPO's contracting efficiency is "low" if $\Delta f \in [0, \Delta f^{(3)}]$, and "high" if $\Delta f \in (\Delta f^{(3)}, +\infty)$. Intuitively, at equilibrium, the manufacturer and the GPO set prices to extract as much profit as possible from the providers, since the providers are identical. This reasoning results in the following theorem.

Theorem 6.1 (Characterization of SPNE, identical providers, with a GPO)**.** *Given different levels of contracting efficiency, the following strategy profiles are SPNE, with their associated payoffs:*

	strategies			*payoffs*		
contracting efficiency	s_i	p^{G}	γ	π_i	π_{G}	π_{M}
"low" $(0 \le \Delta f \le \Delta f^{(3)})$	*GPO*	$\hat{p} + \Delta f/q$	$\gamma^{(3)}$	$\pi_i^{(1)}$	*0*	$\pi_{\mathrm{M}}^{(6)}$
"high" $(\Delta f > \Delta f^{(3)})$	*GPO*	$\hat{p} + \Delta f/q$	*0*	$\pi_i^{(1)}$	$\pi_{\mathrm{G}}^{(3)}$	$\pi_{\mathrm{M}}^{(7)}$

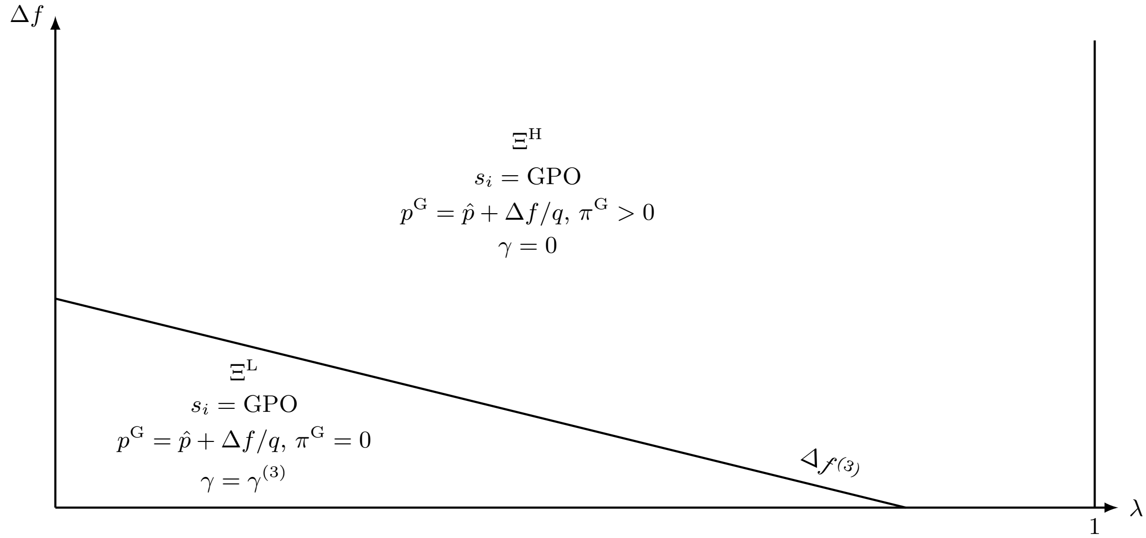

Figure 3: Characterization of the SPNE described in Theorem 6.1 in $(\lambda, \Delta f)$-space.

where

$$\pi_i^{(1)} = q\hat{p} + f^M, \quad \pi_{\mathrm{G}}^{(3)} = nq\big(\hat{p} - (1-\lambda)p^*\big) + n\hat{f}^G + n\Delta f,$$
$$\pi_{\mathrm{M}}^{(6)} = nq\hat{p} + n\hat{f}^G + n\Delta f, \quad \pi_{\mathrm{M}}^{(7)} = (1-\lambda)nqp^*.$$

The managerial interpretations for the equilibrium behavior in the case of two heterogeneous providers discussed in §5.1 hold in this case of n identical providers as well.

As before, we define regions of "low" and "high" contracting efficiency in $(\lambda, \Delta f)$-space:

$$\Xi^{\mathrm{L}} = \{(\lambda, \Delta f) : 0 \le \Delta f \le \Delta f^{(3)},\ 0 \le \lambda \le 1\}, \qquad \Xi^{\mathrm{H}} = \{(\lambda, \Delta f) : \Delta f > \Delta f^{(3)},\ 0 \le \lambda \le 1\}.$$

Figure 3 illustrates the characterization of SPNE described in Theorem 6.1 in $(\lambda, \Delta f)$-space. The providers' costs, the GPO's profit, and the manufacturer's profit at equilibrium as functions of λ and Δf are:

$$\pi_i(\lambda, \Delta f) = \pi_i^{(1)} \quad \text{for all } (\lambda, \Delta f) \in \Xi^{\mathrm{L}} \cup \Xi^{\mathrm{H}},$$
$$\pi_{\mathrm{G}}(\lambda, \Delta f) = \begin{cases} 0 & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{L}}, \\ \pi_{\mathrm{G}}^{(3)} & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{H}}; \end{cases} \qquad \pi_{\mathrm{M}}(\lambda, \Delta f) = \begin{cases} \pi_{\mathrm{M}}^{(6)} & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{L}}, \\ \pi_{\mathrm{M}}^{(7)} & \text{if } (\lambda, \Delta f) \in \Xi^{\mathrm{H}}. \end{cases}$$

We also look at the GPO's profit share ρ^{G} as a function of λ and Δf.

Corollary 6.2. *Suppose f^M and $\hat{f}^G$ are fixed. Then:*

	$\pi_i(\lambda, \Delta f)$	$\pi_{\mathrm{G}}(\lambda, \Delta f)$	$\pi_{\mathrm{M}}(\lambda, \Delta f)$	ρ^G
as a fn. of Δf	*constant*	*nondecreasing, piecewise linear, convex*	*nondecreasing, piecewise linear, concave*	*nondecreasing*
as a fn. of λ	*constant*	*nondecreasing, piecewise linear, convex*	*nonincreasing, piecewise linear, concave*	*nondecreasing*

These behaviors are qualitatively identical to the case of two heterogeneous providers. The top row of plots in Figure 6 (page 28) illustrate the results of Corollary 6.2 in $(\lambda, \Delta f)$-space. The diagonal lines in each plot are provided as a guide for comparison with Figure 3. As before, lighter areas indicate smaller values and darker areas, larger values. Comparing the shading in the top row of Figure 6 with that of Figure 5, we see that the results are quite similar.

Now we turn to equilibrium behavior in the game without a GPO. Using Lemma 4.4, we have the following characterization of subgame perfect Nash equilibrium.

Theorem 6.3 (Characterization of SPNE, identical providers, without a GPO)**.** *The strategy profile* $s_i = mfr$*,* $\gamma = \frac{p^* - \hat{p}}{q}$ *is an SPNE, with associated payoffs* $\pi^i = q\hat{p} + f^M$ *and* $\pi_{\mathrm{M}} = nq\hat{p}$*.*

As in the case of two heterogeneous providers, the equilibria described in Theorem 6.3 are largely driven by the usual trade-off between price and volume. Like the case of two heterogeneous providers, the providers face a *higher* unit price in the presence of a GPO, which is offset by the lower contracting costs in the presence of a GPO.

7 Returning to more general cases

In this section, we identify equilibrium behaviors from §5 and §6 that appear to extend to more general cases: n providers with arbitrary fixed purchasing requirements and a manufacturer's quantity-discount schedule that is nonlinear.

7.1 The case of n providers with arbitrary purchasing requirements and linear quantity discount

Given that all of the providers buy through the GPO in the two special cases examined above (both with a linear quantity-discount schedule), we decided to examine the scenario with n heterogeneous providers and a linear quantity-discount schedule, partly to see if the "all-providers-buy-through-the-GPO" result was an artifact of our model, but more importantly, to see if other characteristics of the equilibria in these special cases continued to apply in this more general scenario.

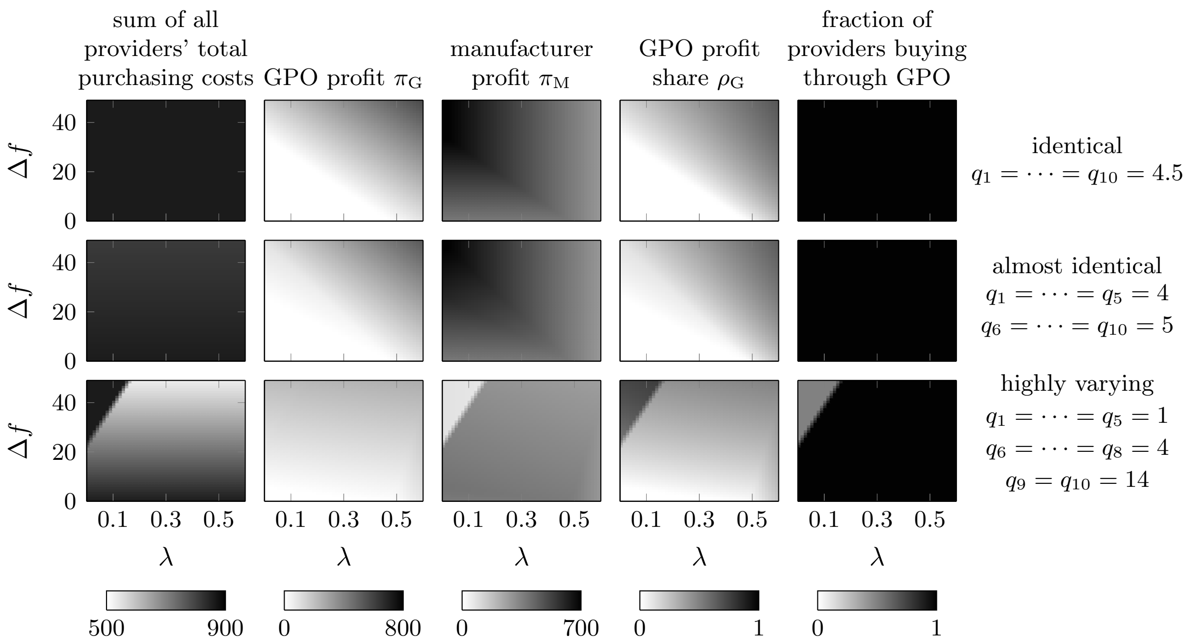

Figure 4: Comparison of equilibrium behavior with 10 providers under different purchasing requirement distributions, with a linear quantity discount schedule. Here, $n = 10$, $p^* = 16$, $\hat{p} = 8$, $\hat{f}^G = 1$, $f^M = 50$.

Although we are unable to obtain closed-form expressions for equilibrium strategies and payoffs in this case, we computed them for a variety of parameterizations, using the procedure described in §4. We discretized the space of linear quantity-discount schedules $p(q) = p^* - \gamma q$ by restricting the domain of γ to 1,000 uniformly spaced values in $[0, \gamma^{\max}]$. Using this discretization of the quantity-discount schedule, we computed approximate SPNEs for 2,500 uniformly spaced points in the $(\lambda, \Delta f)$-space, where $\lambda \in [0, 1/2]$ and $\Delta f \in [0, f^M - \hat{f}^G]$. In this paper, we only show results for $\lambda \in [0, 1/2]$, since values of the CAF λ are typically closer to 0 than 1.

Compare the top row of Figure 4, which shows the equilibrium behavior of an instance of the identical-provider case (from §6), and the middle row, which shows the equilibrium behavior of an instance of the "almost-identical-providers" case. (In both instances, the total provider requirement equals 45.) Note that in the "almost-identical-providers" case, all of the providers purchase their requirements through the GPO in equilibrium, just like in the identical-provider case. In addition, the influence of λ and Δf on GPO and manufacturer profit, and GPO profit share in this "almost-identical provider" case are the same as those in the identical-provider case. For example, in both cases, the GPO's profit is nondecreasing in λ and Δf, while the manufacturer's profit is nonincreasing in λ and nondecreasing in Δf.

On the other hand, a large variance in the providers' purchasing requirements provides the GPO the opportunity to maximize its profit by setting its price so that it attracts only the smallest providers. The bottom row of Figure 4 displays the equilibrium behavior of 10 providers with highly varying purchasing requirements. (Again, the total provider requirement equals 45.) As shown in the last plot of the bottom row, not all the providers buy through the GPO; instead, providers 6 through 10—the providers with the five largest purchasing requirements—buy directly from the competitive source when the GPO's contracting efficiency is high and the CAF is low (the upper-left of the plot) while the providers with smaller purchasing requirements buy through the GPO. Note in the other plots on the bottom row of Figure 4, that this same region of $(\lambda, \Delta f)$-space provides relatively higher profits to the GPO and relatively much lower profits to the manufacturer.

These observations are intuitive. When the providers are relatively homogeneous, they each benefit similarly from the aggregation ability of the GPO. In addition, the homogeneity of the purchasing requirements diminishes the effect of GPO's tradeoff between its unit on-contract price and volume. However, when the providers are relatively heterogeneous, they benefit differently from the aggregating ability of the GPO. Also, when the GPO's contracting efficiency is high, the GPO is in a position to attract virtually any provider that it chooses to. Hence, its profit-maximizing price does not necessarily attract the largest providers.

The following theorem provides sufficient conditions for this observed behavior in the general case: n providers with arbitrary purchasing requirements and a generic manufacturer's quanitty-discount schedule $p(\cdot)$ such that $p(q)$ is nonincreasing in q and the associated revenue $p(q)q$ is nondecreasing in q.

Theorem 7.1. *Suppose $n \geq 3$. In addition, suppose there exists a constant M independent of Δf such that for any $p(\cdot)$ that is feasible for the manufacturer, $0 \leq p(q) \leq M$ for all $q \geq 0$. Let $k' = \min\{k : q_{k+1} = \cdots = q_n\}$. If*

$$\frac{1}{q_n}\sum_{i=1}^{n-1} q_i - \frac{1}{q_{k'}}\sum_{i=1}^{k'-1} q_i < 0, \tag{7.1}$$

then for sufficiently high Δf, there exist providers that do not purchase through the GPO in equilibrium.

In particular, Theorem 7.1 holds when the manufacturer's quantity-discount schedule is linear—that is, of the form (5.1). In this case, since the manufacturer's choice of discount rate γ is constrained between 0 and $\gamma^{\max} = p^*/(2\sum_{i=1}^n q_i)$, the unit price $p(q)$ is bounded, independent of Δf. Note also that (7.1) can never hold when $n = 2$ or with n identical providers.

In summary, in the limited number of instances we observed, the qualitative results for scenarios with 2 heterogeneous providers (§5) and n identical providers (§6) appear to apply to scenarios with n similar, but heterogeneous providers. When there are many providers with highly varying purchasing requirements, all providers no longer necessarily purchase through the GPO.

7.2 Linear vs. nonlinear quantity-discount schedules

In §5 and §6, we studied games where the manufacturer announces a linear quantity-discount schedule of the form (5.1). Now suppose that the manufacturer announces a *nonlinear* quantity-discount schedule of the following form:

$$p(q) = \tilde{p} + \frac{\eta}{q^\gamma}. \tag{7.2}$$

Schotanus et al. (2009) proposed this functional form, tested its fit using "actual offers provided to purchasing groups, and internet stores," and reported that this functional form "fits very well with almost all quantity discount schedule types" they examined. As before, the manufacturer's decision variable is γ. Note that when $\eta > 0$, the quantity-discount schedule $p(q)$ is nonincreasing in q for all q if and only if $\gamma \geq 0$, and the associated revenue $p(q)q$ is nondecreasing in q for all $q \in [0, \sum_{i=1}^n q_i]$ if and only if $\gamma \in [0, 1]$. On the other hand, when $\eta < 0$, the quantity-discount schedule $p(q)$ is nonincreasing in q for all q if and only if $\gamma \leq 0$. In this case, there exists some $\gamma^{\min} \in (-\infty, 0]$ such that the associated revenue $p(q)q$ is nondecreasing in q for all $q \in [0, \sum_{i=1}^n q_i]$ if and only if $\gamma \in [\gamma^{\min}, 0]$.

Although we are unable to obtain a closed-form characterization of the equilibrium strategies and payoffs when the quantity-discount schedule is of the form (7.2), the equilibria can be computed numerically, using the procedure described in §4. For these computations, we fixed the value of η, and discretized the space of nonlinear quantity-discount schedules by restricting the domain of γ. When $\eta > 0$, we restricted the domain of γ to 1,000 uniformly spaced values in $[0, 1]$. When $\eta < 0$, we computed the value of $\gamma^{\min}$ (described above), and restricted the domain of γ to 1,000 uniformly spaced values in $[\gamma^{\min}, 0]$. Using this discretization of the quantity-discount schedules, we computed approximate SPNEs for 2,500 uniformly spaced points in the $(\lambda, \Delta f)$-space, where $\lambda \in [0, 1]$ and $\Delta f \in [0, f^{\mathrm{M}} - \hat{f}^{\mathrm{G}}]$.

Through these computational experiments, we observed that in both the case of two heterogeneous providers and the case of n identical providers, the game with the nonlinear quantity-discount schedule appears to exhibit the same properties as we observed in the game with the linear quantity-discount schedule. In particular, the qualitative attributes in Corollaries 5.2 and 6.2 match. To illustrate: Figure 5 shows the equilibrium behavior for an instance of the game with two heterogeneous providers, under a linear quantity-discount schedule (with $p^* = 16$) and two different nonlinear quantity-discount schedules (one with $\tilde{p} = 0$ and $\eta = 16$, and the other with $\tilde{p} = 16$ and $\eta = -2$). Figure 6 shows the equilibrium behavior for an instance with 10 identical providers, again under a linear and two different nonlinear quantity-discount schedules. Figure 7 shows a comparison of the quantity-discount schedules used in these examples.

Note that the quantity-discount schedules used in these examples are similar, especially in their general behavior (nonincreasing, convex) and range. One might expect that similar sets of available quantity-discount schedules will yield similar equilibrium behavior regardless of the specific functional forms, as we have observed. This apparent robustness to the specific functional form might also be explained by the structural results in Lemmas 4.1-4.3, which hold for *any* quantity discount function that is nonincreasing in quantity and has nondecreasing associated revenue. This includes, for example, stepwise quantity discount functions, which Schotanus et al. (2009) fit using functions of the form (7.2).

In summary, for scenarios with (1) n similar but heterogeneous providers facing a linear quantity-discount schedule, (2) two heterogeneous providers facing a nonlinear quantity-discount schedule, and (3) n identical providers facing a nonlinear quantity-discount schedule, we observe computationally that the behavior of the players' equilibrium payoffs/costs with respect to λ and Δf are similar to those proven for the scenarios with two heterogeneous providers and n identical providers, facing a linear quantity-discount schedule (§5 and §6). Based on this empirical evidence, and the general structural results governing equilibria in Lemmas 4.1-4.3, we conjecture that the results describing the behavior of the players' equilibrium payoffs/costs with respect to λ and Δf can be generalized beyond the cases studied in §5 and §6. Of course, this is only a conjecture, and proof of this conjecture requires further research.

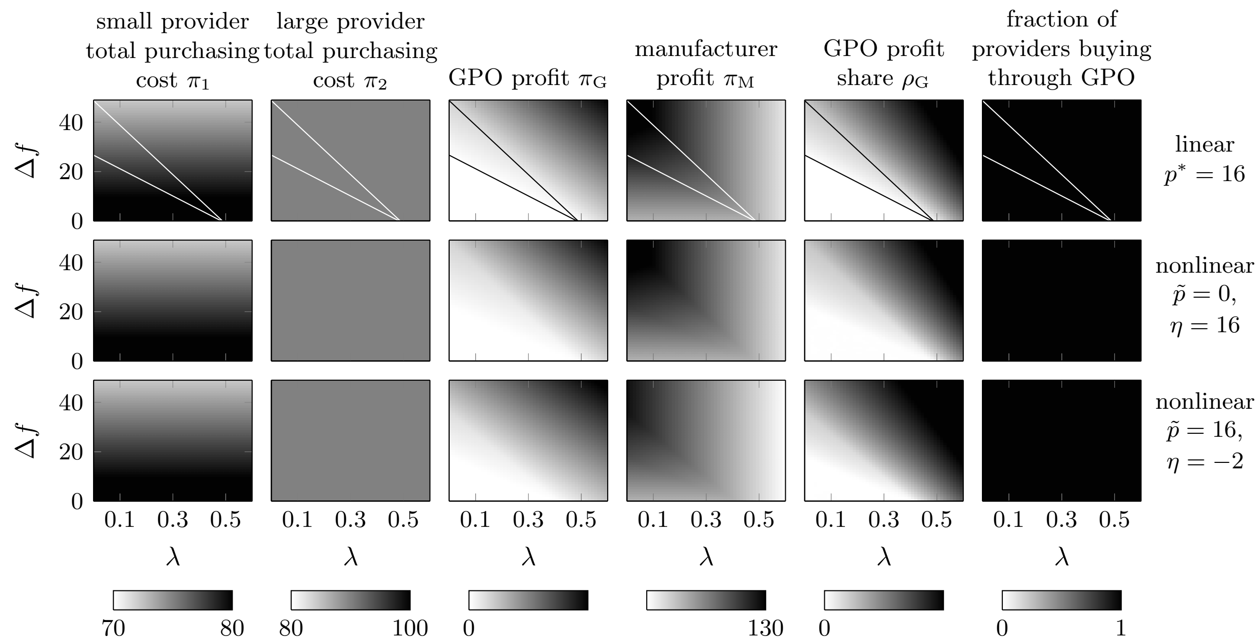

Figure 5: Comparison of equilibrium behavior under different quantity discount schedules, in the case of two heterogeneous providers. Here, $n = 2$, $q_1 = 4$, $q_2 = 5$, $\hat{p} = 8$, $\hat{f}^{\mathrm{G}} = 1$, $f^{\mathrm{M}} = 50$.

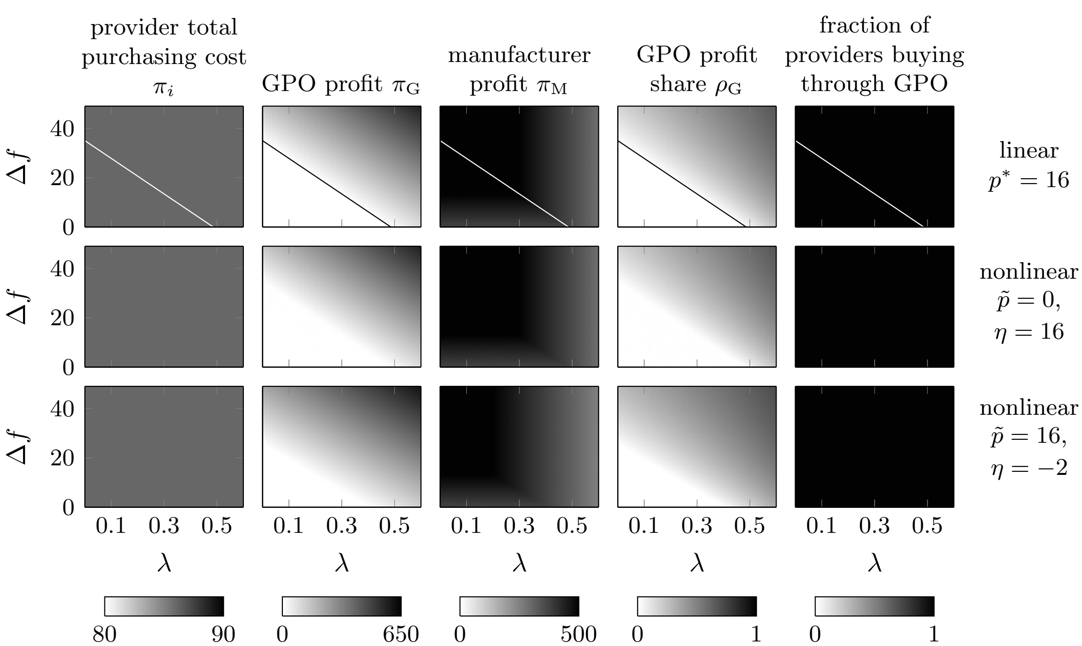

Figure 6: Comparison of equilibrium behavior under different quantity discount schedules, in the case of 10 identical providers. Here, $n = 10$, $q_1 = \cdots = q_{10} = 4.5$, $\hat{p} = 8$, $\hat{f}^{\mathrm{G}} = 1$, $f^{\mathrm{M}} = 50$.

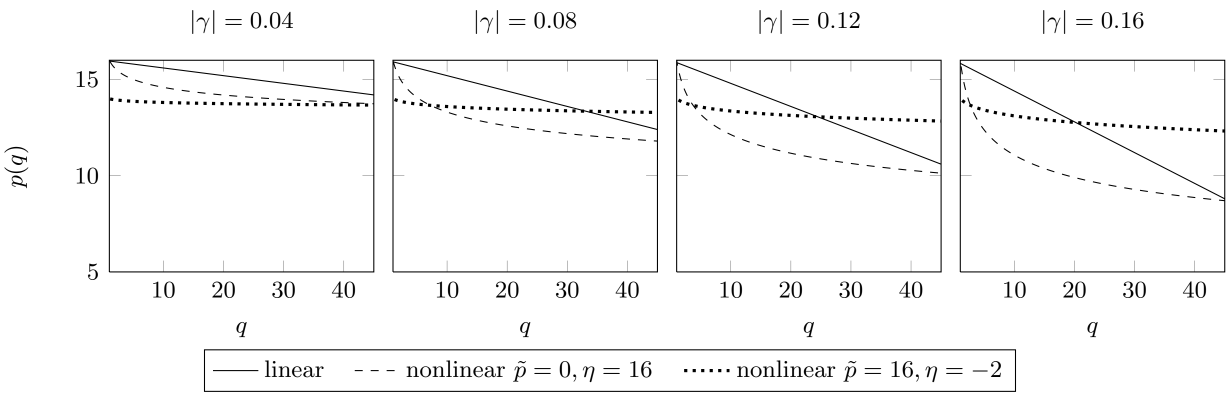

Figure 7: Comparison of quantity discount functions.

8 Concluding remarks

In this section, we answer the five questions posed in the introduction, adding a few suggestions for future research. Before doing so, we acknowledge that our model, and, hence, our results is/are limited to a scenario in which provider demand is inelastic (i.e., fixed provider requirements). Our model is also limited to a single product, whereas GPO pricing sometimes involves bundles of products. These extensions are worthy of future research. We will comment on the impact of other model assumptions below.

Do providers experience lower prices or lower total purchasing costs with a GPO in the supply chain? Based on Lemma 4.2, in the general case, the GPO will set its price to be equal to the breakeven price—the price that equalizes the total purchasing cost—of the largest provider that it chooses to contract for (i.e., at the price that will maximize the GPO's profit). Providers with smaller purchasing requirements will experience lower total purchasing costs in the presence of a GPO, but may experience higher per-unit prices.

These answers must be carefully interpreted when provider-members share in GPO profits. Our model could be modified to account for this by including such profit-sharing in each provider's total purchasing costs, and therefore each provider's breakeven price. This, and the fact that large providers are more likely to be GPO owners, would increase the likelihood that larger providers will purchase through the GPO. This is a topic worthy of future research. Large providers may also demand that the GPO share its CAF. In effect, this would decrease such providers' per unit cost and increase the likelihood of their purchases through the GPO. This, too, deserves more study.

Do CAFs mean higher prices paid by providers? In the two special cases examined, the *total* *purchasing cost* of the providers is not affected by the CAF, although providers may experience higher unit prices. Based on computational experiments, it seems that this behavior occurs in more general cases as well. Interestingly, this matches one of the conclusions of Hu and Schwarz (2011).

How does the presence of the GPO affect manufacturer profits? As displayed in all the cases examined, the manufacturer's profit either does not change or decreases as the GPO's CAF increases. In addition, the manufacturer's profit and profit share either does not change or increases as the GPO's contracting efficiency increases. Thus, the manufacturer benefits partially from the GPO's contracting efficiency. Computational experiments indicate that this occurs in other cases as well.

What affects GPO profits? In the special cases examined, we have demonstrated that GPO profit either does not change or increases as the CAF increases and as the GPO's contracting efficiency decreases. Indeed, for low values of these parameters the GPO makes no profit. It appears that the same behavior holds for more general cases, based on computational tests. In contrast to the non-profit-maximizing GPO studied in Hu and Schwarz (2011), we show that the profit-maximizing GPO in our model does make a profit in some cases.

Recall that in our model, the GPO's contracting efficiency is net of the provider's membership fee; i.e., the higher the membership fee, the lower the GPO's contracting efficiency. Note that GPO membership fees may be different for smaller versus larger providers. Our model could be modified to account for this by adjusting each provider's membership fee, and therefore each provider's breakeven price accordingly. This, too, is a topic for future research.

How are supply-chain profits divided between the manufacturer and the GPO and how is this influenced by the "power" of the GPO? As displayed in all cases examined, the GPO's share of supply-chain profits increases or remains the same as either the CAF or the GPO's contracting efficiency increases. The more powerful the GPO is in negotiating its CAF, and the more efficient it is, the higher its profit and its share of total supply-chain profit.

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A Proofs

We abuse notation and let $\pi^{\mathrm{G}}(p^{\mathrm{G}})$ denote the GPO's profit as a function of its unit price p^{G}, and let $\pi^{\mathrm{M}}(\gamma)$ denote the manufacturer's revenue as a function of its discount rate γ.

A.1 Section 4

Proof of Lemma 4.1. Consider provider $i \in \{1, \dots, n\}$. Suppose provider i purchases non-zero quantities from all of its three options. In particular, suppose provider i purchases $\beta^{\mathrm{G}} q_i$ through the GPO, $\beta^{\mathrm{M}} q_i$ from the manufacturer, and $(1-\beta^{\mathrm{G}}-\beta^{\mathrm{M}})q_i$ from the competitive source, for some $\beta^{\mathrm{G}}, \beta^{\mathrm{M}} \in (0,1)$ such that $\beta^{\mathrm{G}}+\beta^{\mathrm{M}} < 1$. Then, provider i's total purchasing cost is

$$\begin{aligned} f^{\mathrm{G}} &+ \beta^{\mathrm{G}} q_i p^{\mathrm{G}} + f^{\mathrm{M}} + \beta^{\mathrm{M}} q_i (p^* - \gamma\beta^{\mathrm{M}} q_i) + f^{\mathrm{M}} + (1-\beta^{\mathrm{G}}-\beta^{\mathrm{M}} q_i \hat{p}) \\ &\geq \beta^{\mathrm{G}}(f^{\mathrm{G}} + q_i p^{\mathrm{G}}) + \beta^{\mathrm{M}}(f^{\mathrm{M}} + q_i(p^* - \gamma q_i)) + (1-\beta^{\mathrm{G}}-\beta^{\mathrm{M}})(f^{\mathrm{M}} + q_i \hat{p}) \\ &\geq \min\{f^{\mathrm{G}} + q_i p^{\mathrm{G}}, f^{\mathrm{M}} + q_i(p^* - \gamma q_i), f^{\mathrm{M}} + q_i \hat{p}\}. \end{aligned}$$

The first inequality holds since $f^{\mathrm{G}} \geq 0$, $f^{\mathrm{M}} \geq 0$, $p^{\mathrm{G}} \geq 0$, $\hat{p} \geq 0$, and $p(\beta^{\mathrm{M}} q_i) \geq p(q_i)$. As a result, it is clear that for fixed values of p^{G} and γ, it is optimal for provider i to purchase its entire requirement from the option that offers the lowest total cost. The lemma follows from this observation. □

Proof of Lemma 4.2. Recall that we assume $q_1 \leq \cdots \leq q_n$ without loss of generality. Suppose p^{G} is an optimal strategy for the GPO, such that $p^{\mathrm{G}} \leq \min\{p(q_k), \hat{p}\} + \frac{\Delta f}{q_k}$ for some k that satisfies $q_k < q_j$ for all $j = k+1, \dots, n$. Then, by Lemma 4.1, providers $1, \dots, k$ purchase their entire requirement through the GPO, and the GPO's profit is $k\hat{f}^{\mathrm{G}} + p^{\mathrm{G}} \sum_{i=1}^{k} q_i - (1-\lambda)p(\sum_{j=1}^{k} q_j) \sum_{i=1}^{k} q_i$. Since p^{G} is optimal, it must be that $p^{\mathrm{G}} = \min\{p(q_k), \hat{p}\} + \frac{\Delta f}{q_k}$. Note that if $p^{\mathrm{G}} = +\infty$, then the GPO's profit is 0. The claim follows. □

Proof of Lemma 4.3. This follows as a consequence of Lemma 4.2 and backwards induction. □

Proof of Lemma 4.4. Similar to proof of Lemma 4.1. □

Proof of Lemma 4.5. This follows as a consequence of Lemma 4.4 and backwards induction. □

A.2 Section 5

Before we begin, we show some properties of $\Delta f^{(1)}$, $\Delta f^{(2)}$, $\gamma^{(0)}$, $\gamma^{(1)}$, and $\gamma^{(2)}$, where

$$\gamma^{(0)} = \frac{q_1((1-\lambda)p^* - \hat{p}) - \hat{f}^{\mathrm{G}} - \Delta f}{(1-\lambda)q_1^2}.$$

Lemma A.1. (a) $\max\{\Delta f^{(1)}, 0\} \le \max\{\Delta f^{(2)}, 0\}$ *for any* $\lambda \in [0,1]$*;* (b) *If* $\Delta f \le \Delta f^{(1)}$*, then* $\gamma^{(0)} \ge \gamma^{(1)} \ge \gamma^{(2)}$*;* (c) *If* $\Delta f > \Delta f^{(1)}$*, then* $\gamma^{(0)} < \gamma^{(1)} < \gamma^{(2)}$*;* (d) *If* $0 \le \Delta f \le \max\{0, \Delta f^{(1)}\}$*, then* $\gamma^{(1)} \ge 0$*;* (e) $\Delta f \le \Delta f^{(2)}$ *if and only if* $\gamma^{(2)} \ge 0$*;* (f) $\gamma^{(1)} \le \frac{p^* - \hat{p}}{q_2}$*;* (g) $\gamma^{(2)} \le \frac{p^* - \hat{p}}{q_2}$.

Proof. First, we show part a. Let

$$\lambda^{(1)} = \frac{p^* - \hat{p}}{p^*} - \frac{\hat{f}^{\mathrm{G}}}{p^*}\frac{q_2^2 - q_1^2 + 2q_1q_2}{q_1q_2(q_1+q_2)} \quad \text{and} \quad \lambda^{(2)} = \frac{p^* - \hat{p}}{p^*} - \frac{\hat{f}^{\mathrm{G}}}{p^*}\frac{1}{q_2}.$$

Note that $\Delta f^{(1)} \ge 0$ when $\lambda \le \lambda^{(1)}$, and $\Delta f^{(2)} \ge 0$ when $\lambda \le \lambda^{(2)}$. We have that $\lambda^{(2)} \ge \lambda^{(1)}$, since

$$\lambda^{(2)} - \lambda^{(1)} = \frac{\hat{f}^{\mathrm{G}}}{p^*}\big(\frac{q_2^2 - q_1^2 + 2q_1q_2}{q_1q_2(q_1+q_2)} - \frac{1}{q_2}\big) = \frac{\hat{f}^{\mathrm{G}}}{p^*q_2}\frac{q_2(q_2-q_1)}{q_1(q_1+q_2)} \ge 0.$$

Therefore, for all $\lambda \in [\lambda^{(1)}, 1]$, the claim holds. Note that

$$\Delta f^{(1)} = \frac{q_1q_2^2(q_1+q_2)}{q_2^3 - q_1^3 + 2q_1q_2^2}(\lambda^{(1)} - \lambda) \quad \text{and} \quad \Delta f^{(2)} \ge \frac{q_2^2}{q_1}(\lambda^{(1)} - \lambda).$$

Since

$$\frac{q_2^2}{q_1} - \frac{q_1q_2^2(q_1+q_2)}{q_2^3 - q_1^3 + 2q_1q_2^2} = \frac{q_2^2(q_2^3 - q_1^3 + 2q_1q_2^2) - q_1^2q_2^2(q_1+q_2)}{q_1(q_2^3 - q_1^3 + 2q_1q_2^2)} = \frac{q_2^2(q_2^3 - q_1^3 + q_1(2q_2^2 - q_1^2 - q_1q_2))}{q_1(q_2^3 - q_1^3 + 2q_1q_2^2)} \ge 0,$$

for any $\lambda \in [0, \lambda^{(1)}]$, the claim also holds.

Now, we show part b. Since $\Delta f \le \Delta f^{(1)}$, we have that

$$\gamma^{(0)} - \gamma^{(1)} = \frac{q_1q_2^2(q_1+q_2)((1-\lambda)p^* - \hat{p}) - q_2(q_2^2 - q_1^2 + 2q_1q_2)\hat{f}^{\mathrm{G}} - \Delta f(q_2^3 - q_1^3 + 2q_1q_2^2)}{(1-\lambda)q_1^2(q_1+q_2)^2} \ge 0,$$

$$\gamma^{(1)} - \gamma^{(2)} = \frac{q_2(q_1+q_2)(2q_1+q_2)((1-\lambda)p^* - \hat{p}) - 2q_1(2q_1+q_2)\hat{f}^{\mathrm{G}} - \Delta f(1 + \frac{q_1}{q_2})q_2(2q_1+q_2)}{(1-\lambda)q_2(q_1+q_2)^2(2q_1+q_2)}$$
$$- \frac{q_2(q_1+q_2)^2((1-\lambda)p^* - \hat{p}) - (q_1+q_2)^2\hat{f}^{\mathrm{G}} - \Delta f\frac{q_1}{q_2}(q_1+q_2)^2}{(1-\lambda)q_2(q_1+q_2)^2(2q_1+q_2)} \ge 0.$$

Part c follows in a similar manner.

Next, we show part d.

$$\begin{aligned}\gamma^{(1)} &= \frac{(q_1+q_2)((1-\lambda)p^* - \hat{p}) - 2\hat{f}^{\mathrm{G}} - \Delta f(1 + \frac{q_1}{q_2})}{(1-\lambda)(q_1+q_2)^2} \\ &\overset{\text{(i)}}{\ge} \frac{q_2(q_1+q_2)(q_2^2 - q_1^2)((q_1+q_2)(p^* - \hat{p}) - (q_1+q_2)p^*\lambda - \hat{f}^{\mathrm{G}})}{(1-\lambda)(q_1+q_2)^2(q_2^3 - q_1^3 + 2q_1q_2)} \\ &\overset{\text{(ii)}}{\ge} \frac{(q_1+q_2)(q_2^2 - q_1^2)(q_2^2 - q_1^2 + q_1q_2)}{(1-\lambda)q_1q_2(q_1+q_2)^2(q_2^3 - q_1^3 + 2q_1q_2)} \ge 0.\end{aligned}$$

Above, inequality (i) holds since $\Delta f \le \Delta f^{(1)}$, and inequality (ii) holds since $\lambda \le \lambda^{(1)}$.

Part e holds, since $\Delta f \leq \Delta f^{(2)}$ implies

$$\gamma^{(2)} = \frac{q_2((1-\lambda)p^* - \hat{p}) - \hat{f}^{\mathrm{G}} - \Delta f \frac{q_1}{q_2}}{(1-\lambda)q_1^2} \geq \frac{q_2((1-\lambda)p^* - \hat{p}) - \hat{f}^{\mathrm{G}} - q_2((1-\lambda)p^* - \hat{p}) + \hat{f}^{\mathrm{G}}}{(1-\lambda)q_1^2} = 0.$$

Finally, parts f and g hold, since

$$\frac{p^* - \hat{p}}{q_2} - \gamma^{(1)} = \frac{(1-\lambda)q_1(q_1+q_2)(p^* - \hat{p}) + q_2(q_1+q_2)\lambda p^* + 2q_2\hat{f}^{\mathrm{G}} + \Delta f(q_1+q_2)}{(1-\lambda)q_2(q_1+q_2)^2} \geq 0,$$

$$\frac{p^* - \hat{p}}{q_2} - \gamma^{(2)} = \frac{2(1-\lambda)(p^* - \hat{p}) + q_2\lambda\hat{p} + \hat{f}^{\mathrm{G}} + \Delta f \frac{q_1}{q_2}}{(1-\lambda)q_2(2q_1+q_2)} \geq 0. \qquad \square$$

Lemma A.2. *Let γ be any given strategy for the manufacturer.*

a. Given "low" contracting efficiency; i.e., $\Delta f \leq \Delta f^{(1)}$: the optimal strategies of the providers and the GPO are: $s_1 =$ comp, $s_2 =$ comp, $p^{\mathrm{G}} = +\infty$ if $\gamma \in [0, \gamma^{(1)})$; $s_1 =$ GPO, $s_2 =$ GPO, $p^{\mathrm{G}} = \hat{p} + \frac{\Delta f}{q_2}$ if $\gamma \in [\gamma^{(1)}, \frac{p^*-\hat{p}}{q_2})$; $s_1 =$ GPO, $s_2 =$ GPO, $p^{\mathrm{G}} = p^* - \gamma q_2 + \frac{\Delta f}{q_2}$ if $\gamma \in [\frac{p^*-\hat{p}}{q_2}, \gamma^{\max}]$.

b. Given "moderate" contracting efficiency; i.e., $\Delta f^{(1)} < \Delta f \leq \Delta f^{(2)}$: the optimal strategies of the providers and the GPO are: $s_1 =$ comp, $s_2 =$ comp, $p^{\mathrm{G}} = +\infty$ if $\gamma \in [0, \max\{0, \gamma^{(0)}\})$; $s_1 =$ GPO, $s_2 =$ comp, $p^{\mathrm{G}} = \hat{p} + \frac{\Delta f}{q_1}$ if $\gamma \in [\max\{0, \gamma^{(0)}\}, \gamma^{(2)})$; $s_1 =$ GPO, $s_2 =$ GPO, $p^{\mathrm{G}} = \hat{p} + \frac{\Delta f}{q_2}$ if $\gamma \in [\gamma^{(2)}, \frac{p^*-\hat{p}}{q_2})$; $s_1 =$ GPO, $s_2 =$ GPO, $p^{\mathrm{G}} = p^* - \gamma q_2 + \frac{\Delta f}{q_2}$ if $\gamma \in [\frac{p^*-\hat{p}}{q_2}, \gamma^{\max}]$.

c. Given "high" contracting efficiency; i.e., $\Delta f > \Delta f^{(2)}$: the optimal strategies of the providers and the GPO are $s_1 =$ GPO, $s_2 =$ GPO, $p^{\mathrm{G}} = \hat{p} + \frac{\Delta f}{q_2}$ if $\gamma \in [0, \frac{p^*-\hat{p}}{q_2})$; $s_1 =$ GPO, $s_2 =$ GPO, $p^{\mathrm{G}} = p^* - \gamma q_2 + \frac{\Delta f}{q_2}$ if $\gamma \in [\frac{p^*-\hat{p}}{q_2}, \gamma^{\max}]$.

Proof. First, note that since $q_1 < q_2$ and $\Delta f > 0$, we have that for any given $\gamma \geq 0$, $\hat{p} + \frac{\Delta f}{q_2} < \hat{p} + \frac{\Delta f}{q_1}$ and $p^* - \gamma q_2 + \frac{\Delta f}{q_2} < p^* - \gamma q_1 + \frac{\Delta f}{q_1}$. We consider three cases, based on the value of γ.

First, suppose $\gamma \in [0, \frac{p^*-\hat{p}}{q_2})$, or equivalently, $\hat{p} < p^* - \gamma q_2 < p^* - \gamma q_1$. In this case, by Lemma 4.1, we have that: $s_1 =$ GPO and $s_2 =$ GPO if $p^{\mathrm{G}} \in [0, \hat{p} + \frac{\Delta f}{q_2}]$; $s_1 =$ GPO and $s_2 =$ comp if $p^{\mathrm{G}} \in (\hat{p} + \frac{\Delta f}{q_2}, \hat{p} + \frac{\Delta f}{q_1}]$; $s_1 =$ comp and $s_2 =$ comp if $p^{\mathrm{G}} \in (\hat{p} + \frac{\Delta f}{q_1}, +\infty)$. Since the GPO maximizes its profit, its optimal strategy p^{G} must be either $\hat{p} + \frac{\Delta f}{q_2}$, $\hat{p} + \frac{\Delta f}{q_1}$, or $+\infty$. Note that

$$\pi_{\mathrm{G}}(\hat{p} + \tfrac{\Delta f}{q_2}) = 2\hat{f}^{\mathrm{G}} + \Delta f(1 + \tfrac{q_1}{q_2}) - (q_1+q_2)((1-\lambda)p^* - \hat{p}) + (1-\lambda)(q_1+q_2)^2\gamma,$$

$$\pi_{\mathrm{G}}(\hat{p} + \tfrac{\Delta f}{q_1}) = \hat{f}^{\mathrm{G}} + \Delta f - q_1((1-\lambda)p^* - \hat{p}) + (1-\lambda)q_1^2\gamma, \qquad \pi_{\mathrm{G}}(+\infty) = 0.$$

It is straightforward to show that: $\pi_{\mathrm{G}}(\hat{p} + \frac{\Delta f}{q_1}) > \pi_{\mathrm{G}}(+\infty)$ if and only if $\gamma > \gamma^{(0)}$; $\pi_{\mathrm{G}}(\hat{p} + \frac{\Delta f}{q_2}) > \pi_{\mathrm{G}}(+\infty)$ if and only if $\gamma > \gamma^{(1)}$; and $\pi_{\mathrm{G}}(\hat{p} + \frac{\Delta f}{q_2}) > \pi_{\mathrm{G}}(\hat{p} + \frac{\Delta f}{q_1})$ if and only if $\gamma > \gamma^{(2)}$.

Suppose $0 \leq \Delta f \leq f^{(1)}$. By Lemma A.1.ace, then the GPO's optimal strategy p^{G} is: $+\infty$ if

$\gamma \in [0, \gamma^{(1)}]$; $\hat{p} + \frac{f}{q_2}$ if $\gamma \in (\gamma^{(1)}, \frac{p^* - \hat{p}}{q_2}]$. Suppose $\Delta f^{(1)} < \Delta f \leq \Delta f^{(2)}$. Then, by Lemma A.1.bf, the GPO's optimal strategy p^{G} is: $+\infty$ if $\gamma \in [0, \max\{0, \gamma^{(0)}\}]$; $\hat{p} + \frac{\Delta f}{q_1}$ if $\gamma \in (\max\{0, \gamma^{(0)}\}, \gamma^{(2)}]$; $\hat{p} + \frac{\Delta f}{q_2}$ if $\gamma \in (\gamma^{(2)}, \frac{1}{q_2}(p^* - \hat{p})]$. Finally, suppose $\Delta f > \Delta f^{(2)}$. Then, by Lemma A.1.bdf, the GPO's optimal strategy is $p^{\text{G}} = \hat{p} + \frac{\Delta f}{q_2}$ for all $\gamma \in [0, \frac{p^* - \hat{p}}{q_2}]$.

Next, suppose $\gamma \in [\frac{p^* - \hat{p}}{q_2}, \frac{p^* - \hat{p}}{q_1})$, or equivalently, $p^* - \gamma q_2 \leq \hat{p} < p^* - \gamma q_1$. In this case, by Lemma 4.1, we have that: $s_1 =$ GPO and $s_2 =$ GPO if $p^{\text{G}} \in [0, p^* - \gamma q_2 + \frac{\Delta f}{q_2}]$; $s_1 =$ GPO and $s_2 =$ mfr if $p^{\text{G}} \in (p^* - \gamma q_2 + \frac{\Delta f}{q_2}, \hat{p} + \frac{\Delta f}{q_1}]$; $s_1 =$ comp and $s_2 =$ mfr if $p^{\text{G}} \in (\hat{p} + \frac{\Delta f}{q_1}, +\infty)$. Since the GPO maximizes its profit, its optimal strategy p^{G} must be either $p^* - \gamma q_2 + \frac{\Delta f}{q_2}$, $\hat{p} + \frac{\Delta f}{q_1}$, or $+\infty$. The associated GPO profits are

$$\pi_{\text{G}}(p^* - \gamma q_2 + \tfrac{\Delta f}{q_2}) = 2\hat{f}^{\text{G}} + \lambda(q_1 + q_2)(p^* - \gamma(q_1 + q_2)) + \Delta f(1 + \tfrac{q_1}{q_2}) + \gamma q_1(q_1 + q_2),$$
$$\pi_{\text{G}}(\hat{p} + \tfrac{\Delta f}{q_1}) = \hat{f}^{\text{G}} + q_1\hat{p} + \Delta f - (1 - \lambda)q_1(p^* - \gamma q_1), \qquad \pi_{\text{G}}(+\infty) = 0.$$

Since $\gamma \in [0, \gamma^{\max}]$, the expression $q(p^* - \gamma q)$ is increasing in q. Also note that $\hat{p} < p^* - \gamma q_1$. It follows that

$$\begin{aligned}
\pi_{\text{G}}(\hat{p} + \tfrac{\Delta f}{q_1}) &= \hat{f}^{\text{G}} + q_1\hat{p} + \Delta f - (1 - \lambda)q_1(p^* - \gamma q_1) \\
&< \hat{f}^{\text{G}} + \Delta f + q_1(p^* - \gamma q_1) - (1 - \lambda)q_1(p^* - \gamma q_1) = \hat{f}^{\text{G}} + \Delta f + \lambda q_1(p^* - \gamma q_1) \\
&< \pi_{\text{G}}(p^* - \gamma q_2 + \tfrac{\Delta f}{q_2}).
\end{aligned}$$

It is clear that $\pi_{\text{G}}(p^* - \gamma q_2 + \frac{\Delta f}{q_2}) > 0 = \pi_{\text{G}}(+\infty)$. Therefore, the GPO's optimal strategy is $p^{\text{G}} = p^* - \gamma q_2 + \frac{\Delta f}{q_2}$.

Finally, suppose $\gamma \in [\frac{p^* - \hat{p}}{q_1}, \gamma^{\max}]$, or equivalently, $p^* - \gamma q_2 < p^* - \gamma q_1 \leq \hat{p}$. In this case, by Lemma 4.1, we have that: $s_1 =$ GPO and $s_2 =$ GPO if $p^{\text{G}} \in [0, p^* - \gamma q_2 + \frac{\Delta f}{q_2}]$; $s_1 =$ GPO and $s_2 =$ mfr if $p^{\text{G}} \in (p^* - \gamma q_2 + \frac{\Delta f}{q_2}, p^* - \gamma q_1 + \frac{\Delta f}{q_1}]$; $s_1 =$ mfr and $s_2 =$ mfr if $p^{\text{G}} \in (p^* - \gamma q_1 + \frac{\Delta f}{q_1}, +\infty)$. Since the GPO maximizes its profit, its optimal strategy p^{G} must be either $p^* - \gamma q_2 + \frac{\Delta f}{q_2}$, $p^* - \gamma q_1 + \frac{\Delta f}{q_1}$, or $+\infty$. Note that

$$\pi_{\text{G}}(p^* - \gamma q_2 + \tfrac{\Delta f}{q_2}) = 2\hat{f}^{\text{G}} + \lambda(q_1 + q_2)(p^* - \gamma(q_1 + q_2)) + f(1 + \tfrac{q_1}{q_2}) + \gamma q_1(q_1 + q_2),$$
$$\pi_{\text{G}}(p^* - \gamma q_1 + \tfrac{\Delta f}{q_1}) = \hat{f}^{\text{G}} + \lambda q_1(p^* - \gamma q_1) + \Delta f, \qquad \pi_{\text{G}}(+\infty) = 0.$$

Since $\gamma \in [0, \gamma^{\max}]$, the expression $q(p^* - \gamma q)$ is increasing in q. It follows that $\pi_{\text{G}}(p^* - \gamma q_2 + \frac{\Delta f}{q_2}) > \pi_{\text{G}}(p^* - \gamma q_1 + \frac{\Delta f}{q_1}) > \pi_{\text{G}}(+\infty)$, and so the GPO's optimal strategy is $p^{\text{G}} = p^* - \gamma q_2 + \frac{\Delta f}{q_2}$.

Putting the three cases together implies the lemma. □

Proof of Theorem 5.1. We first consider the case of "low" contracting efficiency. Suppose $\Delta f \leq \Delta f^{(1)}$. By Lemma A.2, the manufacturer's revenue $\pi_{\mathrm{M}}(\gamma)$ as a function of γ is: 0 if $\gamma \in [0, \gamma^{(1)})$; $(1-\lambda)(q_1+q_2)(p^* - \gamma(q_1+q_2))$ if $\gamma \in [\gamma^{(1)}, \gamma^{\max}]$. Note that $\pi_{\mathrm{M}}(\gamma)$ is nonincreasing in γ on every interval, and therefore the manufacturer's optimal discount rate must be attained at the left endpoint of one of these intervals: that is, it must be either 0 or $\gamma^{(1)}$. Since

$$\pi_{\mathrm{M}}(\gamma^{(1)}) = (1-\lambda)(q_1+q_2)(p^* - \gamma^{(1)}(q_1+q_2)) = (q_1+q_2)\big(\hat{p} + \frac{\Delta f}{q_2}\big) + 2\hat{f}^{\mathrm{G}} \geq 0 = \pi_{\mathrm{M}}(0),$$

it follows that the strategy profile $s_1 = \mathrm{GPO}$, $s_2 = \mathrm{GPO}$, $p^{\mathrm{G}} = \hat{p} + \Delta f/q_2$, $\gamma = \gamma^{(1)}$ is an SPNE.

Next, we consider the case of "moderate" contracting efficiency. Suppose $\Delta f \in (\Delta f^{(1)}, \Delta f^{(2)}]$. By Lemma A.2, the manufacturer's revenue $\pi_{\mathrm{M}}(\gamma)$ as a function of γ is: 0 if $\gamma \in [0, \max\{0, \gamma^{(0)}\})$; $(1-\lambda)q_1(p^* - \gamma q_1)$ if $\gamma \in [\max\{0,\gamma^{(0)}\}, \gamma^{(2)})$; $(1-\lambda)(q_1+q_2)(p^* - \gamma(q_1+q_2))$ if $\gamma \in [\gamma^{(2)}, \gamma^{\max}]$. Again, note that $\pi_{\mathrm{M}}(\gamma)$ is nonincreasing in γ on every interval, and therefore the manufacturer's optimal discount rate must be attained at either 0, $\max\{0, \gamma^{(0)}\}$, or $\gamma^{(1)}$. We have that

$$\pi_{\mathrm{M}}(\gamma^{(0)}) = q_1\hat{p} + \hat{f}^{\mathrm{G}} + \Delta f, \qquad \pi_{\mathrm{M}}(\gamma^{(2)}) = (q_1+q_2)\big(\tfrac{q_1}{2q_1+q_2}(1-\lambda)p^* + \tfrac{q_1+q_2}{2q_1+q_2}\big(\hat{p} + \tfrac{\hat{f}^{\mathrm{G}}}{q_2} + \tfrac{\Delta f}{q_2}\tfrac{q_1}{q_2}\big)\big).$$

Suppose $\gamma^{(0)} > 0$; that is, $\Delta f < q_1((1-\lambda)p^* - \hat{p}) - \hat{f}^{\mathrm{G}}$ and $\lambda < \frac{p^*-\hat{p}}{p^*} - \frac{\hat{f}^{\mathrm{G}}}{q_1 p^*}$. Then,

$$\begin{aligned}
&\pi_{\mathrm{M}}(\gamma^{(2)}) - \pi_{\mathrm{M}}(\gamma^{(0)}) \\
&\quad = (q_1+q_2)\left(\frac{q_1}{2q_1+q_2}(1-\lambda)p^* + \frac{q_1+q_2}{2q_1+q_2}\left(\hat{p} + \frac{\hat{f}^{\mathrm{G}}}{q_2} + \frac{\Delta f}{q_2}\frac{q_1}{q_2}\right)\right) - q_1\hat{p} - \hat{f}^{\mathrm{G}} - \Delta f \\
&\quad = \frac{q_1(q_1+q_2)}{2q_1+q_2}(1-\lambda)p^* + \frac{q_2^2 - q_1^2 + q_1q_2}{2q_1+q_2}\hat{p} - \frac{q_2^3 - q_1^3 + q_1q_2^2 - 2q_1^2q_2}{q_2^2(2q_1+q_2)}\Delta f + \frac{q_1^2}{q_2(2q_1+q_2)}\hat{f}^{\mathrm{G}} \\
&\quad \overset{\text{(i)}}{>} \frac{q_1^3(q_1+2q_2)}{q_2^2(2q_1+q_2)}(1-\lambda)p^* + \frac{(q_2-q_1)(q_1+q_2)^3}{q_2^2(2q_1+q_2)}\hat{p} + \frac{(q_2-q_1)(q_1+q_2)^2}{q_2^2(2q_1+q_2)}\hat{f}^{\mathrm{G}} \\
&\quad \overset{\text{(ii)}}{>} q_2\hat{p} + \frac{q_1^2+q_2^2+q_1q_2}{q_2(2q_1+q_2)}\hat{f}^{\mathrm{G}} \geq 0,
\end{aligned}$$

where inequality (i) holds because $\Delta f < q_1((1-\lambda)p^* - \hat{p}) - \hat{f}^{\mathrm{G}}$, and inequality (ii) holds because $\lambda < \frac{p^*-\hat{p}}{p^*} - \frac{\hat{f}^{\mathrm{G}}}{q_1p^*}$. Therefore, $\pi_{\mathrm{M}}(\gamma^{(2)}) \geq \pi_{\mathrm{M}}(\gamma^{(0)}) \geq \pi_{\mathrm{M}}(0)$, and so the strategy profile $s_1 = \mathrm{GPO}$, $s_2 = \mathrm{GPO}$, $p^{\mathrm{G}} = \hat{p} + \Delta f/q_2$, $\gamma = \gamma^{(2)}$ is an SPNE.

Now suppose $\gamma^{(0)} \leq 0$; that is, $\Delta f \geq q_1((1-\lambda)p^* - \hat{p}) - \hat{f}^{\mathrm{G}}$. Then, we have that

$$\pi_{\mathrm{M}}(\gamma^{(2)}) - \pi_{\mathrm{M}}(0) = (q_1+q_2)\left(\frac{q_1}{2q_1+q_2}(1-\lambda)p^* + \frac{q_1+q_2}{2q_1+q_2}\left(\hat{p} + \frac{\hat{f}^{\mathrm{G}}}{q_2} + \frac{\Delta f}{q_2}\frac{q_1}{q_2}\right)\right) - (1-\lambda)q_1p^*$$

$$= -\frac{q_1^2}{2q_1+q_2}(1-\lambda)p^* + \frac{(q_1+q_2)^2}{2q_1+q_2}\left(p^* + \frac{\hat{f}^{\mathrm{G}}}{q_2} + \frac{\Delta f}{q_2}\frac{q_1}{q_2}\right)$$
$$\geq \frac{q_1^3(q_1+2q_2)}{q_2^2(2q_1+q_2)}(1-\lambda)p^* + \frac{(q_2-q_1)(q_1+q_2)^3}{q_2^3(2q_1+q_2)}\hat{p} + \frac{(q_2-q_1)(q_1+q_2)^2}{q_2^2(2q_1+q_2)}\hat{f}^{\mathrm{G}} > 0,$$

and so the strategy profile $s_1 = \mathrm{GPO}$, $s_2 = \mathrm{GPO}$, $p^{\mathrm{G}} = \hat{p} + \Delta f/q_2$, $\gamma = \gamma^{(2)}$ is an SPNE.

Finally, suppose $\Delta f > \Delta f^{(2)}$. By Lemma A.2, the manufacturer's revenue as a function of γ is $\pi_{\mathrm{M}}(\gamma) = (1-\lambda)(q_1+q_2)(p^* - \gamma(q_1+q_2))$ for all $\gamma \in [0, \gamma^{\max}]$. Therefore, in this case, the manufacturer's optimal discount rate is 0, and so the strategy profile $s_1 = \mathrm{GPO}$, $s_2 = \mathrm{GPO}$, $p^{\mathrm{G}} = \hat{p} + \Delta f/q_2$, $\gamma = 0$ is an SPNE. □

Proof of Corollary 5.2. This follows by taking partial derivatives with respect to λ and Δf, and comparing the partial derivatives for $(\lambda, \Delta f)$ in Ξ^{L}, Ξ^{M}, and Ξ^{H}. □

Proof of Theorem 5.3. By Lemma 4.4, the manufacturer's revenue $\pi_{\mathrm{M}}(\gamma)$ as a function of γ is: 0 if $\gamma \in [0, \frac{p^*-\hat{p}}{q_2})$; $q_2(p^* - \gamma q_2)$ if $\gamma \in [\frac{p^*-\hat{p}}{q_2}, \frac{p^*-\hat{p}}{q_1})$; $q_1(p^* - \gamma q_1) + q_2(p^* - \gamma q_2)$ if $\gamma \in [\frac{p^*-\hat{p}}{q_1}, \gamma^{\max}]$. Therefore, the manufacturer's optimal discount rate must be attained at either 0, $\frac{p^*-\hat{p}}{q_2}$, or $\frac{p^*-\hat{p}}{q_1}$. In this case, we have that $\pi_{\mathrm{M}}(0) = 0$, $\pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q_2}) = q_2\hat{p}$, and $\pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q_1}) = q_1\hat{p} + q_2\big(\frac{q_2}{q_1}\hat{p} - \big(\frac{q_2}{q_1} - 1\big)p^*\big)$. Note that $\frac{q_2}{q_1}\hat{p} - \big(\frac{q_2}{q_1} - 1\big)p^* > 0$, since $\gamma^{\max} \geq \frac{p^*-\hat{p}}{q_1}$.

First, we consider the case of "high" competition. Suppose $0 \leq \hat{p} \leq \hat{p}^{(1)}$. Then, $\pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q_2}) - \pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q_1}) = (q_2 - q_1)\hat{p} - q_2\big(\frac{q_2}{q_1}\hat{p} - \big(\frac{q_2}{q_1} - 1\big)p^*\big) = \frac{1}{q_1}\big(q_2(q_2-q_1)p^* - (q_1^2 + q_2(q_2-q_1))\hat{p}\big) \leq 0$. It follows that $\pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q_1}) \geq \pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q_2}) > \pi_{\mathrm{M}}(0)$, and so the strategy profile $s_1 = \mathrm{comp}$, $s_2 = \mathrm{mfr}$, $\gamma = (p^* - \hat{p})/q_2$ is an SPNE.

Next, we consider the case of "low" competition. Suppose $\hat{p}^{(1)} < \hat{p} \leq p^*$. Then, we have that $\pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q_2}) > \pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q_1}) > \pi_{\mathrm{M}}(0)$, and so the strategy profile $s_1 = \mathrm{mfr}$, $s_2 = \mathrm{mfr}$, $\gamma = (p^* - \hat{p})/q_1$ is an SPNE. □

A.3 Section 6

First, we show some properties of $\gamma^{(3)}$ and $\Delta f^{(3)}$.

Lemma A.3. (a) $\gamma^{(3)} \leq \frac{p^*-\hat{p}}{q}$; (b) $\Delta f^{(3)} \geq 0$ *for all* $\lambda \leq \lambda^{(3)} = \frac{p^*-\hat{p}}{p^*} - \frac{\hat{f}^G}{qp^*}$.

Proof. Part a holds, since

$$\frac{p^*-\hat{p}}{q} - \gamma^{(3)} = \frac{(n-1)q(1-\lambda)(p^*-\hat{p}) + \lambda q\hat{p} + \hat{f}^{\mathrm{G}} + \Delta f}{(1-\lambda)nq^2} \geq 0.$$

Part b holds, since $\Delta f^{(3)} = q(p^* - \hat{p}) - \hat{f}^{\mathrm{G}} - qp^*\lambda \geq q(p^* - \hat{p}) - \hat{f}^{\mathrm{G}} - qp^*\lambda^{(3)} = 0$. □

Next, we characterize the optimal strategies of the providers and the GPO as a function of the manufacturer's discount rate γ.

Lemma A.4. *Let γ be any given strategy for the manufacturer.*

a. *Given "low" contracting efficiency—i.e., $\Delta f \leq \Delta f^{(3)}$—the optimal strategies of the providers and the GPO are: $s_i =$ comp, $p^{\mathrm{G}} = +\infty$ if $\gamma \in [0, \gamma^{(3)})$; $s_i =$ GPO, $p^{\mathrm{G}} = \hat{p} + \frac{\Delta f}{q}$ if $\gamma \in [\gamma^{(3)}, \frac{p^*-\hat{p}}{q})$; $s_i =$ GPO, $p^{\mathrm{G}} = p^* - \gamma q + \frac{\Delta f}{q}$ if $\gamma \in [\frac{p^*-\hat{p}}{q}, \gamma^{\max}]$.*

b. *Given "high" contracting efficiency—i.e., $\Delta f > \Delta f^{(3)}$—the optimal strategies of the providers and the GPO are: $s_i =$ GPO, $p^{\mathrm{G}} = \hat{p} + \frac{\Delta f}{q}$ if $\gamma \in [0, \frac{p^*-\hat{p}}{q})$; $s_i =$ GPO, $p^{\mathrm{G}} = p^* - \gamma q + \frac{\Delta f}{q}$ if $\gamma \in [\frac{p^*-\hat{p}}{q}, \gamma^{\max}]$.*

Proof. We consider two cases, based on the level of γ.

First, suppose $\gamma \in [0, \frac{p^*-\hat{p}}{q})$, or equivalently, $\hat{p} < p^* - \gamma q$. In this case, by Lemma 4.1, we have that: $s_i =$ GPO if $p^{\mathrm{G}} \in [0, \hat{p} + \frac{\Delta f}{q}]$; $s_i =$ comp if $p^{\mathrm{G}} \in (\hat{p} + \frac{\Delta f}{q}, +\infty)$. Therefore, the GPO's optimal unit on-contract price p^{G} must be either $\hat{p} + \frac{\Delta f}{q}$ or $+\infty$. Note that

$$\pi_{\mathrm{G}}(\hat{p} + \tfrac{\Delta f}{q}) = n\hat{f}^{\mathrm{G}} + n\Delta f - nq((1-\lambda)p^* - \hat{p}) + (1-\lambda)(nq)^2\gamma, \qquad \pi_{\mathrm{G}}(+\infty) = 0.$$

If $0 \leq \Delta f < \Delta f^{(3)}$, it is straightforward to show that $\pi_{\mathrm{G}}(\hat{p} + \frac{\Delta f}{q}) > \pi_{\mathrm{G}}(+\infty)$ if and only if $\gamma > \gamma^{(3)}$. Therefore, the GPO's optimal unit on-contract price p^{G} is: $+\infty$ if $\gamma \in [0, \gamma^{(3)}]$; $\hat{p} + \frac{\Delta f}{q}$ if $\gamma \in (\gamma^{(3)}, \frac{p^*-\hat{p}}{q}]$. Otherwise, if $\Delta f \geq \Delta f^{(3)}$, we have that that $\pi_{\mathrm{G}}(\hat{p}+\frac{\Delta f}{q}) \geq (1-\lambda)(nq)^2\gamma \geq \pi_{\mathrm{G}}(+\infty)$, for all $\gamma \in [0, \frac{p^*-\hat{p}}{q})$. Therefore, in this case, the GPO's optimal unit on-contract price is $p^{\mathrm{G}} = \hat{p} + \frac{\Delta f}{q}$ for all $\gamma \in [0, \frac{p^*-\hat{p}}{q})$.

Next, suppose $\gamma \in [\frac{p^*-\hat{p}}{q}, \gamma^{\max}]$, or equivalently, $p^* - \gamma q \leq \hat{p}$. In this case, by Lemma 4.1, we have that: $s_i =$ GPO if $p^{\mathrm{G}} \in [0, p^* - \gamma q + \frac{\Delta f}{q}]$; $s_i =$ comp if $p^{\mathrm{G}} \in (p^* - \gamma q + \frac{\Delta f}{q}, +\infty)$. Therefore, the GPO's optimal unit on-contract price p^{G} must be either $p^* - \gamma q + \frac{\Delta f}{q}$ or $+\infty$. Note that $\pi_{\mathrm{G}}(p^* - \gamma q + \frac{\Delta f}{q}) = n\hat{f}^{\mathrm{G}} + n\Delta f + \lambda(nq)(p^* - \gamma q) + (1-\lambda)n(n-1)q^2\gamma \geq 0 = \pi_{\mathrm{G}}(+\infty)$. It follows in this case that the GPO's optimal unit on-contract price is $p^{\mathrm{G}} = p^* - \gamma q + \frac{\Delta f}{q}$ for all $\gamma \in [\frac{p^*-\hat{p}}{q}, \gamma^{\max}]$. □

Proof of Theorem 6.1. We first consider the case of "low" contracting efficiency. Suppose $0 \leq \Delta f \leq \Delta f^{(3)}$. By Lemma A.4, the manufacturer's revenue $\pi_{\mathrm{M}}(\gamma)$ as a function of γ is: 0 if $\gamma \in [0, \gamma^{(3)})$;

$(1-\lambda)(nq)(p^* - \gamma nq)$ if $\gamma \in [\gamma^{(3)}, \gamma^{\max}]$. Note that $\pi_{\mathrm{M}}(\gamma)$ is nonincreasing in γ on every interval, and therefore the manufacturer's optimal discount rate must be attained at the left endpoint of one of these intervals: that is, it must be either 0 or $\gamma^{(3)}$. Since $\pi_{\mathrm{M}}(\gamma^{(3)}) = (1-\lambda)(nq)(p^* - \gamma^{(3)} nq) = nq + n\hat{f}^{\mathrm{G}} + n\Delta f > 0 = \pi_{\mathrm{M}}(0)$, it follows that the strategy profile $s_i = \mathrm{GPO}$, $p^{\mathrm{G}} = \hat{p} + \Delta f/q$, $\gamma = \gamma^{(3)}$ is an SPNE.

Next, we consider part b. By Lemma A.4, the manufacturer's revenue as a function of γ is $\pi_{\mathrm{M}}(\gamma) = (1-\lambda)nqp^*$, and so in this case, the manufacturer's optimal discount rate is 0. Therefore, the strategy profile $s_i = \mathrm{GPO}$, $p^{\mathrm{G}} = \hat{p} + \Delta f/q$, $\gamma = 0$ is an SPNE. □

Proof of Corollary 6.2. This follows by taking partial derivatives with respect to λ and Δf, and comparing the partial derivatives for $(\lambda, \Delta f)$ in Ξ^{L} and Ξ^{H}. □

Proof of Theorem 6.3. By Lemma 4.4, the manufacturer's revenue $\pi_{\mathrm{M}}(\gamma)$ as a function of γ is: 0 if $\gamma \in [0, \frac{p^*-\hat{p}}{q})$; $(nq)(p^* - \gamma q)$ if $\gamma \in [\frac{p^*-\hat{p}}{q}, \gamma^{\max}]$. Therefore, the manufacturer's optimal discount rate must be attained at either 0 or $\frac{p^*-\hat{p}}{q}$. We have that $\pi_{\mathrm{M}}(\frac{p^*-\hat{p}}{q}) = nq\hat{p} \geq 0 = \pi_{\mathrm{M}}(0)$. Therefore, the strategy profile $s_i = \mathrm{mfr}$, $\gamma = (p^* - \hat{p})/q$ is an SPNE. □

A.4 Section 7

Proof of Theorem 7.1. Fix some discount schedule $p(\cdot)$. The breakeven price that induces providers $1, \dots, n$ to purchase through the GPO is $p_n^{\mathrm{B}} = \min\{p(q_n), \hat{p}\} + \frac{\Delta f}{q_n}$. Similarly, the breakeven price that induces providers $1, \dots, k^*$ to purchase through the GPO is $p_{k^*}^{\mathrm{B}} = \min\{p(q_{k^*}), \hat{p}\} + \frac{\Delta f}{q_{k^*}}$. It follows that

$$\begin{aligned}\pi_{\mathrm{G}}(p_n^{\mathrm{B}}) - \pi_{\mathrm{G}}(p_{k^*}^{\mathrm{B}}) &= (n-k^*)\hat{f}^{\mathrm{G}} + \sum_{i=1}^{n} q_i \left(\min\{p(q_n), \hat{p}\} - (1-\lambda) p\left(\sum_{j=1}^{n} q_j \right) \right) \\ &\quad - \sum_{i=1}^{k^*} q_i \left(\min\{p(q_{k^*}), \hat{p}\} - (1-\lambda) p\left(\sum_{j=1}^{k^*} q_j \right) \right) + \Delta f \left(\frac{1}{q_n} \sum_{i=1}^{n-1} q_i - \frac{1}{q_{k^*}} \sum_{i=1}^{k^*-1} q_i \right).\end{aligned}$$

So, if Δf is sufficiently high, by (7.1), we have that $\pi_{\mathrm{G}}(p_n^{\mathrm{B}}) < \pi_{\mathrm{G}}(p_{k^*}^{\mathrm{B}})$ for *any* feasible $p(\cdot)$, since $p(\cdot)$ is bounded from above and below by a constant that is independent of Δf. Therefore, if Δf is sufficiently high, regardless of the manufacturer's choice of discount schedule $p(\cdot)$, the GPO is more profitable if it sets its unit price to obtain the business of providers $1, \dots, k^*$, instead of all of the providers $1, \dots, n$. □