Learning and Time-Varying Macroeconomic Volatility

Fabio Milani
University of California, Irvine

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Introduction

- Strong evidence of changes in macro volatility over time (The Great Moderation)
- Kim and Nelson (1999), McConnell and Pérez-Quiròs (2000), Stock and Watson (2002), Blanchard and Simon (2001)

Time-Varying Volatility

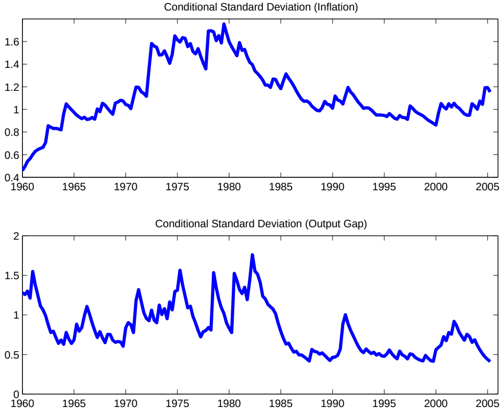

Figure: Conditional Standard Deviation series for Inflation and Output Gap

Introduction

- Need to correctly model volatility
- Sims and Zha (AER 2006): BVAR, Regime changes in volatilities of shocks

Introduction

- In DSGE Models?
  Exogenous shocks with constant variance
  (Smets and Wouters *JEEA* 2003, *AER* 2007, An and Schorfheide *ER* 2007)
- DSGE with Stochastic Volatility
  Justiniano and Primiceri (*AER* forth.), Fernandez-Villaverde and Rubio-Ramirez (*RES* 2007)
- Time variation in the volatility of exogenous shocks

Introduction

- But what explains the changing volatility?

Scope of the paper

- Present a simple model with learning
- The learning speed (gain coefficient) of the agents is endogenous: it responds to previous forecast errors
- Endogenous Time-Varying Volatility
- Related: Branch and Evans (*RED* 2007), Lansing (2007), Bullard and Singh (2007).

Results:

1. The changing gain induces endogenous time variation in the volatilities of the macroeconomic variables the agents try to learn
2. Evidence of time variation in endogenous gain from estimated model
3. The econometrician can spuriously find evidence of stochastic volatility if learning is not taken into account

The Model

- Stylized New Keynesian Model

$$\begin{aligned} \pi_t &= \beta \widehat{E}_t \pi_{t+1} + \kappa x_t + u_t & (1) \\ x_t &= \widehat{E}_t x_{t+1} - \sigma(i_t - \widehat{E}_t \pi_{t+1}) + g_t & (2) \\ i_t &= \rho_t i_{t-1} + (1 - \rho_t)(\chi_{\pi,t}\pi_{t-1} + \chi_{x,t} x_{t-1}) + \varepsilon_t & (3) \end{aligned}$$

- Learning instead of RE
- TV Monetary Policy

Expectations Formation

- VAR to form inflation and output expectations
- Perceived Law of Motion (VAR(1)):

$$Z_t = a_t + b_t Z_{t-1} + \eta_t \tag{4}$$

where $Z_t \equiv [\pi_t, x_t, i_t]'$

- $\approx$ Minimum State Variable solution

Learning

- Coefficient Updating

$$\begin{aligned} \widehat{\phi}_t &= \widehat{\phi}_{t-1} + g_{t,y} R_t^{-1} X_t (Z_t - X_t' \widehat{\phi}_{t-1}) && (5) \\ R_t &= R_{t-1} + g_{t,y} (X_{t-1} X_{t-1}' - R_{t-1}) && (6) \end{aligned}$$

where $\widehat{\phi}_t = (a_t', vec(b_t)')'$ and $X_t \equiv \{1, Z_{t-1}\}_0^{t-1}$.

Endogenous Time-Varying Gain

- Decreasing Gain if Forecast Errors are small
- Switch to Constant Gain if Forecast Errors become large

$$g_{t,y} = \begin{cases} t^{-1} & \text{if } \frac{\sum_{j=0}^{J}(|y_{t-j} - E_{t-j-1}y_{t-j}|)}{J} < \upsilon_t^y \\ \overline{g}_y & \text{if } \frac{\sum_{j=0}^{J}(|y_{t-j} - E_{t-j-1}y_{t-j}|)}{J} \geq \upsilon_t^y, \end{cases} \quad (7)$$

where $y = \pi$, x, i. (Decr. Gain reset to $\frac{1}{\overline{g}_y^{-1} + t}$)

- Similar to Marcet-Nicolini (υ_t is m.a.d. of forecast errors)
- Constant Gain is estimated
- Which situations?

Questions:

1. Does the gain coefficient affect volatility? Can the model generate time-varying volatility in inflation and in the output gap?
2. Does the model fit U.S. data? Is there evidence of changes in the gain over time?
3. Does the omission of learning imply that researchers spuriously find stochastic volatility in the structural shocks?
4. Does the model-implied stochastic volatility resemble the SV estimated from the data?
5. What are the effects of MP on the estimated Volatility?

1. Endogenous Gain and TV Volatility

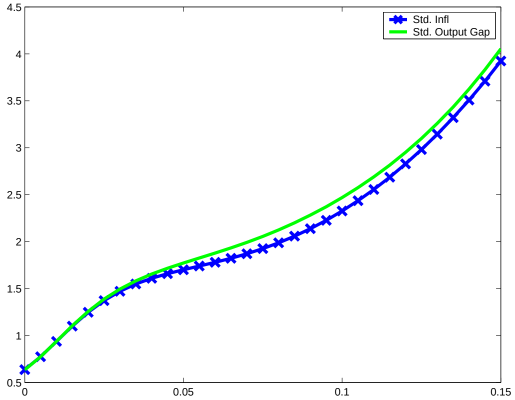

Figure: Volatility of simulated Inflation and Output Gap as a function of the constant gain coefficient.

1. Endogenous Gain and TV Volatility

- Volatility typically increases in the gain
- Simulation (10,000 periods)
- Gain switches endogenously according to previous forecast errors

1. Endogenous Gain and TV Volatility

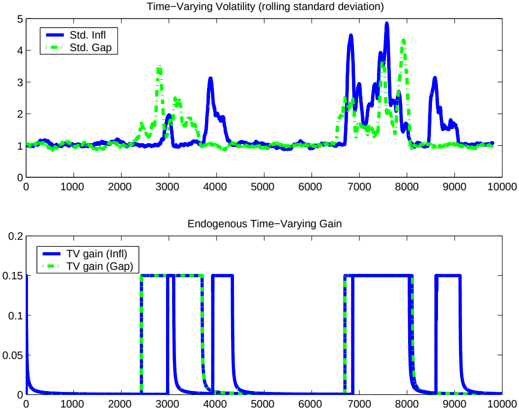

Figure: Time-Varying Volatility with Time-Varying Endogenous Gain Coefficient.

2. Bayesian Estimation

- Gain switches from decreasing to constant
- Constant Gain jointly estimated in the system
- Metropolis-Hastings
- Quarterly U.S. data, 1960:I-2006:I, data from 1954 to 1959 to initialize learning algorithm
- Uniform priors for gains

2. Bayesian Estimation: Priors

		Prior Distribution			
Description	Param.	Range	Distr.	Mean	95% Int.
Inverse IES	σ^{-1}	$\mathbb{R}^+$	G	1	$[.12, 2.78]$
Slope PC	κ	$\mathbb{R}^+$	G	.25	$[.03, .7]$
Discount Rate	β	.99	–	.99	–
Interest-Rate Smooth	ρ_{pre79}	$[0, 1]$	B	.8	$[.46, .99]$
Feedback to Infl.	$\chi_{\pi,pre79}$	$\mathbb{R}$	N	1.5	$[.51, 2.48]$
Feedback to Output	$\chi_{x,pre79}$	$\mathbb{R}$	N	.5	$[.01, .99]$
Interest-Rate Smooth	ρ_{post79}	$[0, 1]$	B	.8	$[.46, .99]$
Feedback to Infl.	$\chi_{\pi,post79}$	$\mathbb{R}$	N	1.5	$[.51, 2.48]$
Feedback to Output	$\chi_{x,post79}$	$\mathbb{R}$	N	.5	$[.01, .99]$
Std. MP shock	σ_{ε}	$\mathbb{R}^+$	IG	1	$[.34, 2.81]$
Std. g_t	σ_g	$\mathbb{R}^+$	IG	1	$[.34, 2.81]$
Std. u_t	σ_u	$\mathbb{R}^+$	IG	1	$[.34, 2.81]$
Constant Gain infl.	$\bar{\mathbf{g}}_{\pi}$	$[0, 0.3]$	U	.15	$[.007, .294]$
Constant Gain gap	$\bar{\mathbf{g}}_{x}$	$[0, 0.3]$	U	.15	$[.007, .294]$
Constant Gain FFR	$\bar{\mathbf{g}}_{i}$	$[0, 0.3]$	U	.15	$[.007, .294]$

Table 1 - Prior Distributions.

2. Bayesian Estimation: Results

		Posterior Distribution	
Description	Parameter	Mean	95% Post. Prob. Int.
Inverse IES	σ^{-1}	6.04	[4.17-9.14]
Slope PC	κ	0.021	[0.0026-0.054]
Discount Factor	β	0.99	-
IRS pre-79	ρ_{pre79}	0.937	[0.85-0.99]
Feedback Infl. pre79	$\chi_{\pi,pre-79}$	1.30	[0.83-1.81]
Feedback Gap pre79	$\chi_{x,pre-79}$	0.66	[0.29-1.13]
IRS post-79	ρ_{post79}	0.93	[0.88-0.97]
Feedback Infl. post79	$\chi_{\pi,post-79}$	1.66	[1.19-2.11]
Feedback Gap post79	$\chi_{x,post-79}$	0.48	[0.07-0.85]
Autoregr. Cost-push shock	ρ_u	0.39	[0.27-0.49]
Autoregr. Demand shock	ρ_g	0.85	[0.78-0.92]
Std. Cost-push shock	σ_u	0.89	[0.81-0.98]
Std. Demand shock	σ_g	0.65	[0.59-0.72]
Std. MP shock	σ_ε	0.97	[0.88-1.07]
Constant gain (Infl.)	$\overline{\mathbf{g}}_\pi$	**0.082**	[0.078-0.09]
Decreasing gain (Infl.)	t^{-1}	-	-
Constant gain (Gap)	$\overline{\mathbf{g}}_x$	**0.073**	[0.06-0.082]
Decreasing gain (Gap)	t^{-1}	-	-
Constant gain (FFR)	$\overline{\mathbf{g}}_i$	0.003	[0,0.023]
Decreasing gain (FFR)	t^{-1}	-	-

Table 2 - Posterior Distributions: baseline case with $J = 4$.

2. Bayesian Estimation: Time-Varying Gain

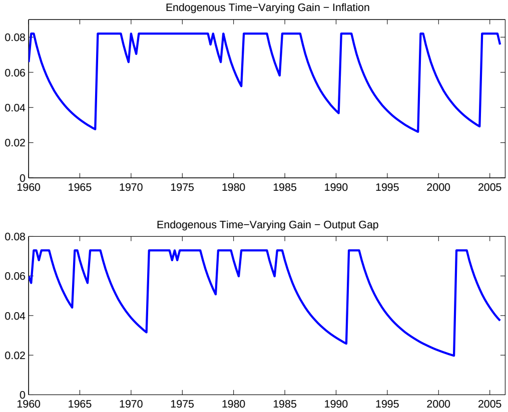

Figure: Endogenous Time-Varying Gain Coefficients (estimated constant gain). Baseline Case

Is it a good idea to use this learning rule?

- Is it dominated by alternatives?

	Endogenous TV Gain	Decreasing Gain	Constant Gain
Inflation	0.94	0.97	0.98
Output Gap	0.88	1.00	0.91

Table 6 - RMSEs.

- Optimality Tests.

$$I_{t+1,t} \equiv \mathbf{1}(Y_{t+1,t} < \widehat{Y}_{t+1,t}) = \alpha + \beta \widehat{Y}_{t+1,t} + u_{t+1} \quad (8)$$

- Back out Loss Function

2. Bayesian Estimation: Time-Varying Gain

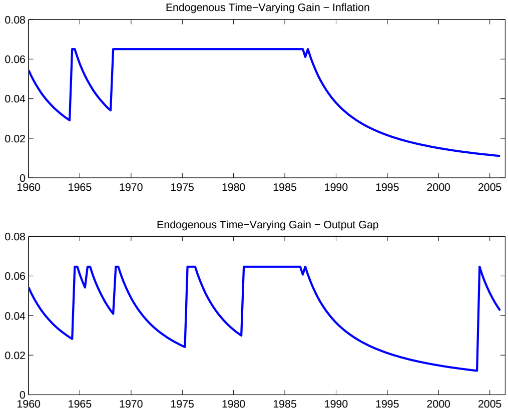

Figure: Endogenous Time-Varying Gain Coefficients (estimated constant gain). Case with $J = 20$

2. Bayesian Estimation: Time-Varying Gain

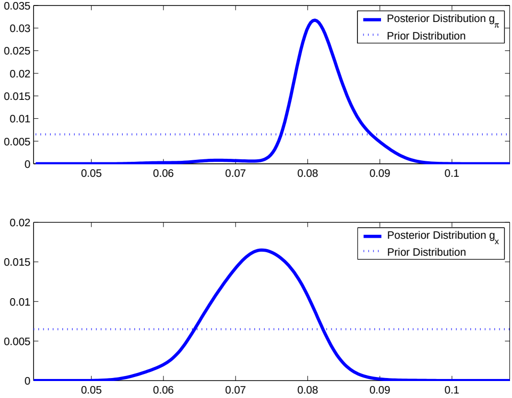

Figure: Constant Gain Coefficients: Prior and Posterior Distributions.

2. Bayesian Estimation: Time-Varying Gain

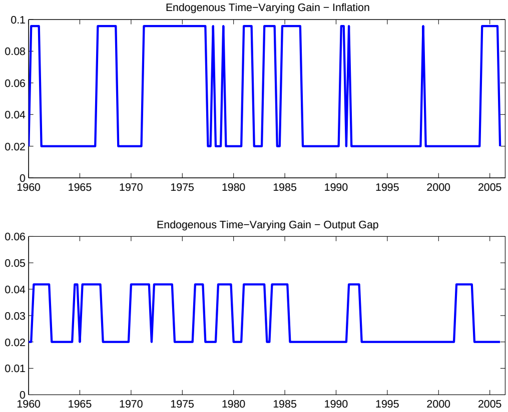

Figure: Endogenous Time-Varying Gain Coefficients (Case with low and high constant gain coefficients only).

2. Bayesian Estimation: Forecast Errors

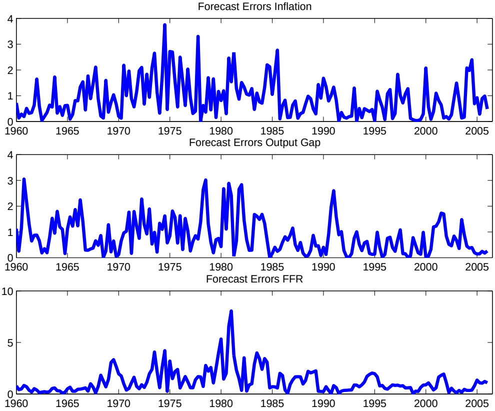

Figure: Forecast errors for inflation, output gap, and federal funds rate (absolute values).

2. Bayesian Estimation: Forecast Errors

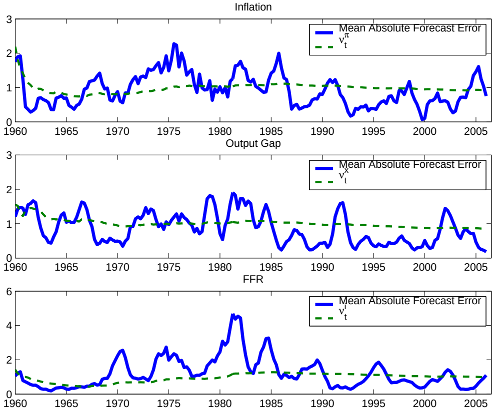

Figure: Rolling Mean Absolute Forecast errors vs. Updated ν_t for inflation, output gap, and federal funds rate series.

3. If learning is neglected:

- The volatility of shocks may be overestimated
- Possible to spuriously find Stochastic Volatility

3. Test for ARCH/GARCH Effects

	Endogenous TV Gain				No Learning	
	$J = 4$		$J = 20$			
	ARCH(1)	GARCH(1,1)	ARCH(1)	GARCH(1,1)	ARCH(1)	GARCH(1,1)
Inflation	0.517	0.61	0.48	0.56	0.05	0.06
Output Gap	0.785	0.89	0.85	0.90	0.045	0.05

Table 7 - Test for the existence of ARCH/GARCH effects (5% significance): proportion of rejections of the null hypothesis of no ARCH/GARCH effects.

4. Volatility

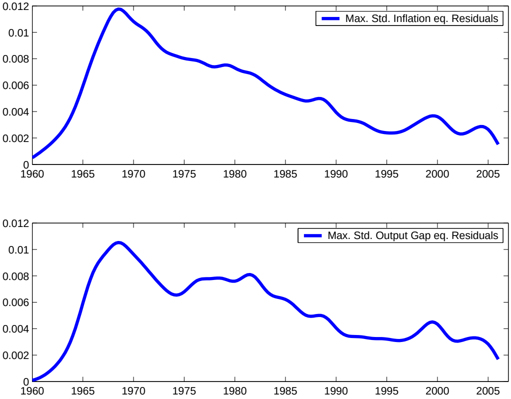

Figure: Maximum rolling Standard Deviation of residuals across simulations: Kernel Density Estimation.

4. The Great Moderation

	Endogenous TV Gain			No Learning	Data
	Baseline	$J = 20$	CG		
Ratio $\frac{Std.\ Infl.\ 1985-2006}{Std.\ Infl.\ 1960-1984}$	0.39	0.42	0.43	1.00	**0.35**
Ratio $\frac{(Std.\ OutputGap\ 1985-2006)}{(Std.\ Output\ Gap\ 1960-1984)}$	0.42	0.52	0.54	1.00	**0.50**

Table 8 - The Great Moderation: ratio of standard deviations for inflation and output gap in the second versus the first part of the simulated samples (median across simulations).

5. Monetary Policy, Learning, and Volatility

- Simulation for $\chi_\pi = [0, ..., 5]$:
- Related: Benati-Surico (2007)

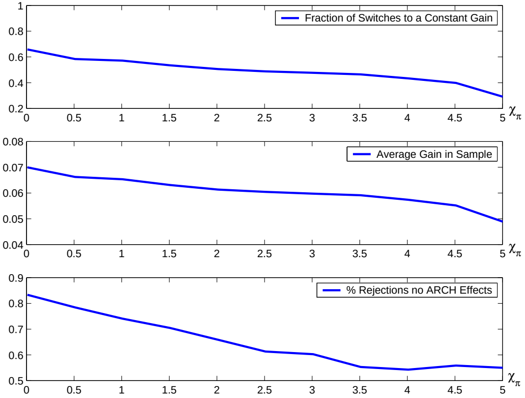

Figure: Effects of Monetary Policy on Volatility.

5. Bernanke - Great Moderation Speech

I am not convinced that the decline in macroeconomic volatility of the past two decades was primarily the result of good luck.

changes in monetary policy could conceivably affect the size and frequency of shocks hitting the economy, at least as an econometrician would measure those shocks

changes in inflation expectations, which are ultimately the product of the monetary policy regime, can also be confused with truly exogenous shocks in conventional econometric analyses.

some of the effects of improved monetary policies may have been misidentified as exogenous changes in economic structure or in the distribution of economic shocks.

6. TV Volatility: Learning or Exogenous Shocks?

- Test ARCH/GARCH in DSGE Model Innovations now

	Output Gap	Inflation
DSGE-RE	ARCH	ARCH
DSGE-TV Gain	ARCH	No ARCH

6. TV Volatility: Learning or Exogenous Shocks?

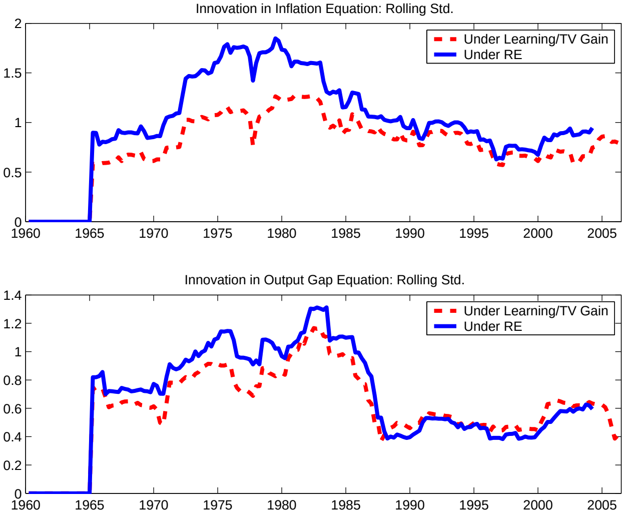

Figure: Rolling Std. estimated innovations under RE and Learning

Conclusions

- Strong Evidence of Stochastic Volatility in the economy
  Usually Exogenous
- Learning with endogenous TV gain (depends on previous forecast errors) $\Rightarrow$ Endogenous Stochastic Volatility
- Gain often larger in pre-1984 sample
- Overestimation of TV in volatility of exogenous shocks.

Future Directions

- How much volatility can learning explain? (estimate DSGE model with learning and TV volatility).
- More serious attempt to match volatility series in the data.
- Different ways to model endogenous gain/ Optimality
- Interactions Policy/Learning/Volatility