Experimental Evaluation of Causal Machine Learning

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Causal Machine Learning for Novel Settings Workshop
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Motivation

- Two revolutions over the past 20 years:
  1. causal inference
  2. machine learning

- Causal machine learning
  1. individualized treatment rules
  2. heterogeneous treatment effects

- Experimental evaluation of causal machine learning (ML)
  - ML algorithms do not necessarily work well in practice
  - uncertainty quantification is important and yet difficult
  - evaluate causal ML before putting it in practice

Evaluating Individualized Treatment Rules

- Individualized treatment rules (ITRs)
  - designed to increase efficiency of policies or treatments
  - personalized medicine, micro-targeting in business/politics

- Existing literature:
  1. estimation of heterogeneous treatment effects
  2. active development of optimal ITRs
  3. extensive use of ML algorithms

- Goal: use a randomized experiment to *evaluate generic ITRs*
  1. use a separate experiment to evaluate ITRs developed with other data
  2. use the same experiment to construct and evaluate ITRs
- Imai and Li. "Experimental Evaluation of Individualized Treatment Rules." *Journal of the American Statistical Association*, Forthcoming.

Key Contributions

1. Neyman's repeated sampling framework
   - random treatment assignment, random sampling
   - no modeling assumption or asymptotic approximation
   - extend analysis to cross-fitting: random splitting

2. Evaluation measures
   - shortcomings of existing metrics
   - incorporating a budget constraint
   - overall evaluation metric for general ITRs

Evaluation without a Budget Constraint

- Setup
  - Binary treatment: $T_i \in \{0, 1\}$
  - Pre-treatment covariates: $\mathbf{X} \in \mathcal{X}$
  - No interference: $Y_i(T_1 = t_1, T_2 = t_2, \ldots, T_n = t_n) = Y_i(T_i = t_i)$
  - Random sampling of units:

$$(Y_i(1), Y_i(0), \mathbf{X}_i) \stackrel{\text{i.i.d.}}{\sim} \mathcal{P}$$

  - Completely randomized treatment assignment:

$$\Pr(T_i = 1 \mid Y_i(1), Y_i(0), \mathbf{X}_i) = \frac{n_1}{n} \quad \text{where} \quad n_1 = \sum_{i=1}^{n} T_i$$

- Fixed (for now) ITR:

$$f : \mathcal{X} \longrightarrow \{0, 1\}$$

  - based on any ML algorithm or even a heuristic rule
  - sample splitting for experimental data, separate observational data

Neyman's Inference for the Standard Metric

- Standard metric (Population Average "Value" or PAV):

$$\lambda_f = \mathbb{E}\{Y_i(f(X_i))\}$$

- A natural estimator:

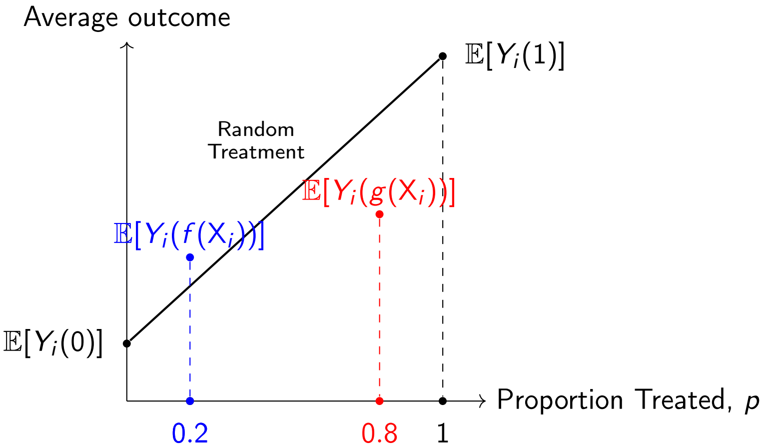
$$\hat{\lambda}_f(\mathcal{Z}) = \frac{1}{n_1} \underbrace{\sum_{i=1}^{n} Y_i T_i f(X_i)}_{\substack{\text{treated units who should}\\ \text{be treated}}} + \frac{1}{n_0} \underbrace{\sum_{i=1}^{n} Y_i(1-T_i)(1-f(X_i))}_{\substack{\text{untreated units who should}\\ \text{not be treated}}},$$

  where $\mathcal{Z} = \{X_i, T_i, Y_i\}_{i=1}^n$
- Unbiasedness: $\mathbb{E}\{\hat{\lambda}_f(\mathcal{Z})\} = \lambda_f$
- Usual variance:

$$\mathbb{V}\{\hat{\lambda}_f(\mathcal{Z})\} = \frac{\mathbb{E}(S_{f1}^2)}{n_1} + \frac{\mathbb{E}(S_{f0}^2)}{n_0},$$

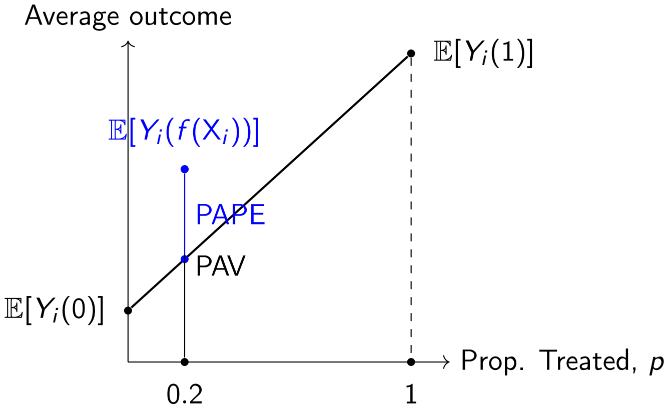
  where $S_{ft}^2 = \sum_{i=1}^n (Y_{fi}(t) - \overline{Y_f(t)})^2/(n-1)$,
  $Y_{fi}(t) = 1\{f(X_i) = t\} Y_i(t)$, and $\overline{Y_f(t)} = \sum_{i=1}^n Y_{fi}(t)/n$ for $t = \{0, 1\}$

A Problem of Comparing ITRs Using the PAV



- $\lambda_f < \lambda_g$: but g is performing worse than the random (i.e., non-individualized) treatment rule whereas f is not
- Need to account for the proportion treated

Accounting for the Proportion of Treated Units



- Population Average Prescriptive Effect (PAPE):

$$\tau_f = \mathbb{E}\{Y_i(f(X_i)) - p_f Y_i(1) - (1 - p_f) Y_i(0)\}$$

where $p_f = \Pr(f(X_i) = 1)$ is the proportion treated under f

Estimating the Population Average Prescriptive Effect

- An unbiased estimator of PAPE τ_f:

$$\hat{\tau}_f(\mathcal{Z}) = \frac{n}{n-1}\underbrace{\left[\frac{1}{n_1}\sum_{i=1}^{n} Y_i T_i f(\mathrm{X}_i) + \frac{1}{n_0}\sum_{i=1}^{n} Y_i(1-T_i)(1-f(\mathrm{X}_i))\right.}_{\text{PAV of ITR}}$$
$$\underbrace{\left. - \frac{\hat{p}_f}{n_1}\sum_{i=1}^{n} Y_i T_i - \frac{1-\hat{p}_f}{n_0}\sum_{i=1}^{n} Y_i(1-T_i)\right]}_{\substack{\text{PAV of random treatment rule with} \\ \text{the same treated proportion}}}$$

  where $\hat{p}_f = \sum_{i=1}^{n} f(\mathrm{X}_i)/n$

- We also derive its variance, and propose its consistent estimator
- Not invariant to additive transformation: $Y_i + c$
- Solution: centering $\mathbb{E}(Y_i(1) + Y_i(0)) = 0 \rightsquigarrow$ minimum variance

Estimating and Evaluating ITRs via Cross-Fitting

- Estimate and evaluate an ITR using the same experimental data
- How should we account for both estimation uncertainty and evaluation uncertainty under the Neyman's framework?
- Setup:
  - Learning algorithm
    $$F : \mathcal{Z} \longrightarrow \mathcal{F}$$
  - K-fold cross-fitting: $\mathcal{Z} = \{\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_K\}$
    $$\hat{f}_{-k} \; = \; F(\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_{k-1}, \mathcal{Z}_{k+1}, \ldots, \mathcal{Z}_K)$$
  - Evaluation metric estimators:
    $$\hat{\lambda}_F \; = \; \frac{1}{K}\sum_{k=1}^{K} \hat{\lambda}_{\hat{f}_{-k}}(\mathcal{Z}_k), \quad \hat{\tau}_F \; = \; \frac{1}{K}\sum_{k=1}^{K} \hat{\tau}_{\hat{f}_{-k}}(\mathcal{Z}_k)$$
- Uncertainty over both <u>evaluation data</u> and <u>all random sets of training data (of a fixed size)</u> as well as treatment assignment

Causal Estimands

- Population Average Value (PAV)
  - Generalized ITR averaging over the random sampling of training data $\mathcal{Z}^{tr}$ (due to random splitting)

$$\bar{f}_F(\mathbf{x}) \;=\; \mathbb{E}\{\hat{f}_{\mathcal{Z}^{tr}}(\mathbf{x}) \mid \mathbf{X}_i = \mathbf{x}\} \;=\; \Pr(\hat{f}_{\mathcal{Z}^{tr}}(\mathbf{x}) = 1 \mid \mathbf{X}_i = \mathbf{x})$$

  - Estimand

$$\lambda_F \;=\; \mathbb{E}\left\{\bar{f}_F(\mathbf{X}_i) Y_i(1) + (1 - \bar{f}_F(\mathbf{X}_i)) Y_i(0)\right\}$$

- Population Average Prescriptive Effect (PAPE)
  - Proportion treated

$$p_F \;=\; \mathbb{E}\{\bar{f}_F(\mathbf{X}_i)\}.$$

  - Estimand

$$\tau_F \;=\; \mathbb{E}\{\lambda_F - p_F Y_i(1) - (1 - p_F) Y_i(0)\}.$$

Inference under Cross-Fitting

- Under Neyman's framework, the cross-fitting estimators are unbiased, i.e., $\mathbb{E}(\hat{\lambda}_F) = \lambda_F$ and $\mathbb{E}(\hat{\tau}_F) = \tau_F$
- The variance of the PAV estimator

$$\mathbb{V}(\hat{\lambda}_F) = \underbrace{\frac{\mathbb{E}(S^2_{\hat{f}1})}{m_1} + \frac{\mathbb{E}(S^2_{\hat{f}0})}{m_0}}_{\text{evaluation uncertainty}} + \underbrace{\mathbb{E}\left\{\mathrm{Cov}(\hat{f}_{\mathcal{Z}^{tr}}(\mathbf{X}_i), \hat{f}_{\mathcal{Z}^{tr}}(\mathbf{X}_j) \mid \mathbf{X}_i, \mathbf{X}_j)\tau_i\tau_j\right\}}_{\text{estimation uncertainty}} - \underbrace{\frac{K-1}{K}\mathbb{E}(S^2_F)}_{\substack{efficiency\ \ gain\ \ due \\ to\ \ cross-fitting}}$$

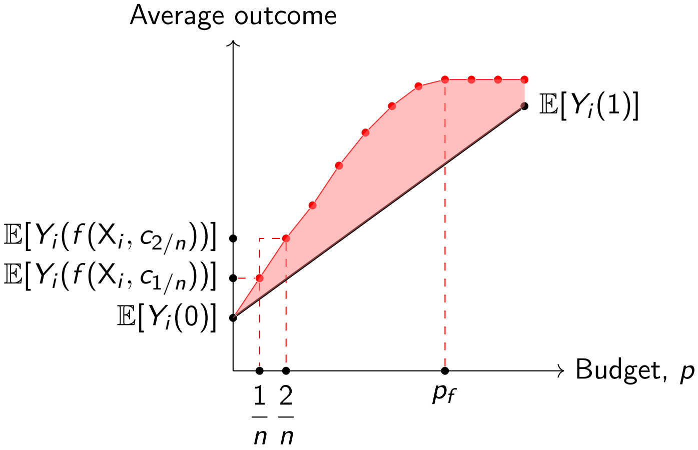
for $i \neq j$ where m_t is the size of the training set with $T_i = t$, $\tau_i = Y_i(1) - Y_i(0)$, $S^2_F = \sum_{k=1}^{K}\left\{\hat{\lambda}_{\hat{f}_{-k}}(\mathcal{Z}_k) - \overline{\hat{\lambda}_{\hat{f}_{-k}}(\mathcal{Z}_k)}\right\}^2/(K-1)$

- Analogous results for the PAPE τ_F

Evaluation with a Budget Constraint

- Policy makers often face a binding budget constraint p
- Scoring rule:
$$s : \mathcal{X} \longrightarrow \mathcal{S} \quad \text{where} \quad \mathcal{S} \subset \mathbb{R}$$
- Example: CATE $s(\mathrm{x}) = \mathbb{E}(Y_i(1) - Y_i(0) \mid \mathrm{X}_i = \mathrm{x})$
- (Fixed) ITR with a budget constraint:
$$f(\mathrm{X}_i, c) \;=\; 1\{s(\mathrm{X}_i) > c\},$$
where $c_p(f) \;=\; \inf\{c \in \mathbb{R} : \Pr(f(\mathrm{X}_i, c) = 1) \leq p\}$
- PAPE under a budget constraint
$$\tau_{fp} \;=\; \mathbb{E}\{Y_i(f(\mathrm{X}_i, c_p(f))) - pY_i(1) - (1-p)Y_i(0)\}.$$
- We derive the bias (and its finite sample bound) and variance under the Neyman's framework
- Extensions: cross-fitting, diff. in PAPE between two ITRs

The Area Under Prescriptive Effect Curve (AUPEC)



- Measure of performance across different budget constraints
- We show how to do inference with and without cross-fitting
- Normalized AUPEC = average percentage gain using an ITR over the randomized treatment rule across a range of budget contraints

Simulations

- Atlantic Causal Inference Conference data analysis challenge
- Data generating process
  - 8 covariates from the Infant Health and Development Program (originally, 58 covariates and 4,302 observations)
  - population distribution = original empirical distribution
  - Model

$$Y_i(t) \;=\; \mu(\mathsf{X}_i) + \tau(\mathsf{X}_i)t + \sigma(\mathsf{X}_i)\epsilon_i,$$

    where $t = 0, 1$, $\epsilon_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, 1)$, and

$$\begin{aligned} \mu(\mathsf{x}) &= -\sin(\Phi(\pi(\mathsf{x}))) + x_{43}, \\ \pi(\mathsf{x}) &= 1/[1 + \exp\{3(x_1 + x_{43} + 0.3(x_{10} - 1)) - 1\}], \\ \tau(\mathsf{x}) &= \xi(x_3 x_{24} + (x_{14} - 1) - (x_{15} - 1)), \\ \sigma(\mathsf{x}) &= 0.25\sqrt{\mathbb{V}(\mu(\mathsf{x}) + \pi(\mathsf{x})\tau(\mathsf{x}))}. \end{aligned}$$

- Two scenarios: large vs. small treatment effects $\xi \in \{2, 1/3\}$
- Sample sizes: $n \in \{100, 500, 2,000\}$

Results I: Fixed ITR

- No budget constraint, 20% constraint
- f: Bayesian Additive Regression Tree (BART)
- g: Causal Forest
- h: LASSO

		$n = 100$			$n = 500$			$n = 2000$		
Estimator	truth	cov.	bias	s.d.	cov.	bias	s.d.	cov.	bias	s.d.
Small effect										
$\hat{\tau}_f$	0.066	94.3	0.005	0.124	96.2	0.001	0.053	95.1	0.001	0.026
$\hat{\tau}_f(c_{0.2})$	0.051	93.2	−0.002	0.109	94.4	0.001	0.046	95.2	0.002	0.021
$\widehat{\Gamma}_f$	0.053	95.3	0.001	0.106	95.1	0.001	0.045	94.8	−0.001	0.024
$\widehat{\Delta}_{0.2}(f, g)$	−0.022	94.0	0.006	0.122	95.4	0.002	0.051	96.0	0.000	0.026
$\widehat{\Delta}_{0.2}(f, h)$	−0.014	93.9	−0.001	0.131	94.9	−0.000	0.060	95.3	−0.000	0.030
Large effect										
$\hat{\tau}_f$	0.430	94.7	−0.000	0.163	95.7	0.000	0.064	94.4	-0.000	0.031
$\hat{\tau}_f(c_{0.2})$	0.356	94.7	0.004	0.159	95.7	0.002	0.072	95.8	0.000	0.035
$\widehat{\Gamma}_f$	0.363	94.3	−0.005	0.130	94.9	0.003	0.058	95.7	0.000	0.029
$\widehat{\Delta}_{0.2}(f, g)$	−0.000	96.9	0.008	0.151	97.9	−0.002	0.073	98.0	−0.000	0.026
$\widehat{\Delta}_{0.2}(f, h)$	0.000	94.7	−0.004	0.140	97.7	−0.001	0.065	96.6	0.000	0.033

Results II: Estimated ITR

- 5-fold cross fitting
- F: LASSO
- std. dev. for $n = 500$ is roughly half of the fixed $n = 100$ case

	$n = 100$			$n = 500$			$n = 2000$		
Estimator	cov.	bias	s.d.	cov.	bias	s.d.	cov.	bias	s.d.
Small effect									
$\hat{\lambda}_F$	96.4	0.001	0.216	96.7	0.002	0.100	97.2	0.002	0.046
$\hat{\tau}_F$	94.6	−0.002	0.130	95.5	−0.002	0.052	94.4	−0.000	0.027
$\hat{\tau}_F(c_{0.2})$	95.4	−0.003	0.120	95.4	−0.002	0.043	96.8	0.001	0.029
$\widehat{\Gamma}_F$	98.2	0.002	0.117	96.8	−0.001	0.048	95.9	0.001	0.001
Large effect									
$\hat{\lambda}_H$	96.9	−0.007	0.261	96.5	−0.003	0.125	97.3	0.001	0.062
$\hat{\tau}_F$	93.6	−0.000	0.171	93.0	0.000	0.093	95.3	0.001	0.041
$\hat{\tau}_F(c_{0.2})$	94.8	−0.002	0.170	96.2	−0.005	0.075	95.8	0.001	0.037
$\widehat{\Gamma}_F$	98.5	0.001	0.126	98.9	0.005	0.053	99.0	0.001	0.026

Application to the STAR Experiment

- Experiment involving 7,000 students across 79 schools
- Randomized treatments (kindergarden):
  1. $T_i = 1$: small class (13–17 students)
  2. $T_i = 0$: regular class (22–25)
  3. regular class with aid
- Outcome: SAT scores
- Literature on heterogeneous treatments in labor economics
- 10 covariates
  - 4 demographics: gender, race, birth month, birth year
  - 6 school characteristics: urban/rural, enrollment size, grade range, number of students on free lunch, percentage white, number of students on school buses
- Sample size: $n = 1,911$, 5-fold cross-fitting
- Average Treatment Effects:
  - SAT reading: 6.78 (s.e.=1.71)
  - SAT math: 5.78 (s.e.=1.80)

Results I: ITR Performance

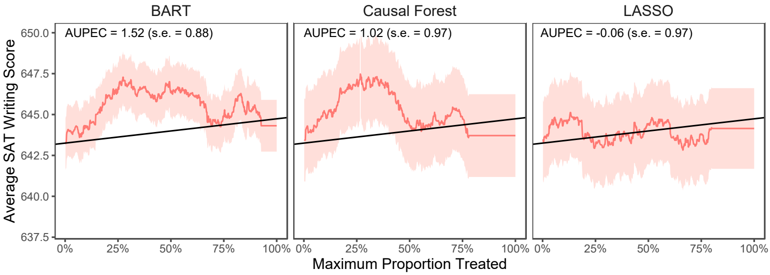
	BART			Causal Forest			LASSO		
	est.	s.e.	treated	est.	s.e.	treated	est.	s.e.	treated
Fixed ITR									
No budget constraint									
Reading	0	0	100%	−0.38	1.14	84.3%	−0.41	1.10	84.4%
Math	0.52	1.09	86.7	0.09	1.18	80.3	1.73	1.25	78.7
Writing	−0.32	0.72	92.7	−0.70	1.18	78.0	−0.30	1.26	80.0
Budget constraint									
Reading	−0.89	1.30	20	0.66	1.23	20	−1.17	1.18	20
Math	0.70	1.25	20	2.57	1.29	20	1.25	1.32	20
Writing	2.60	1.17	20	2.98	1.18	20	0.28	1.19	20
Estimated ITR									
No budget constraint									
Reading	0.19	0.37	99.3%	0.31	0.77	86.6%	0.32	0.53	87.6%
Math	0.92	0.75	84.7	2.29	0.80	79.1	1.52	1.60	75.2
Writing	1.12	0.86	88.0	1.43	0.71	67.4	0.05	1.37	74.8
Budget constraint									
Reading	1.55	1.05	20	0.40	0.69	20	−0.15	1.41	20
Math	2.28	1.15	20	1.84	0.73	20	1.50	1.48	20
Writing	2.31	0.66	20	1.90	0.64	20	−0.47	1.34	20

Results II: Comparison between ML Algorithms

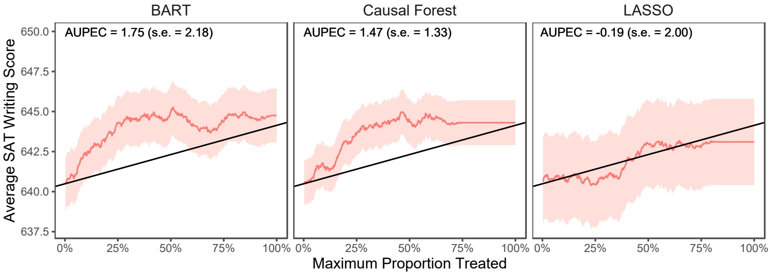
	Causal Forest				BART	
	vs. **BART**		vs. **LASSO**		vs. **LASSO**	
	est.	95% CI	est.	95% CI	est.	95% CI
Fixed ITR						
Math	1.55	[−0.35, 3.45]	1.83	[−0.50, 4.16]	0.28	[−2.39, 2.95]
Reading	1.86	[−0.79, 4.51]	1.31	[−1.49, 4.11]	−0.55	[−4.02, 2.92]
Writing	0.38	[−1.66, 2.42]	2.69	[−0.27, 5.65]	2.32	[−0.53, 5.15]
Estimated ITR						
Reading	−1.15	[−3.99, 1.69]	0.55	[−1.05, 2.15]	1.70	[−0.90, 4.30]
Math	−0.43	[−2.57, 3.43]	0.34	[−1.32, 2.00]	0.77	[−1.99, 3.53]
Writing	−0.41	[−1.63, 0.80]	2.37	[0.76, 3.98]	2.79	[1.32, 4.26]

Results III: AUPEC

Fixed ITR



Estimated ITR



Evaluation of Heterogeneous Treatment Effects

- Another popular use of ML in causal inference
- Estimation of heterogeneous treatment effects: random forest, BART, Lasso, etc.

- How can we make valid inference for heterogeneous treatment effects discovered via a generic ML algorithm?
  - cannot assume ML algorithms converge uniformly
  - avoid computationally intensive method (e.g., repeated cross-fitting)
  - use Neyman's repeated sampling framework for inference

Setup and Causal Quantities of Interest

- Conditional Average Treatment Effect (CATE):

$$\tau(\mathrm{x}) \;=\; \mathbb{E}(Y_i(1) - Y_i(0) \mid \mathrm{X}_i = \mathrm{x})$$

- CATE estimation based on ML algorithm

$$s : \mathcal{X} \longrightarrow \mathcal{S} \subset \mathbb{R}$$

- Sorted Group Average Treatment Effect (GATE; Chernozhukov et al. 2019)

$$\tau_k \;:=\; \mathbb{E}(Y_i(1) - Y_i(0) \mid c_{k-1}(s) \leq s(\mathrm{X}_i) < c_k(s))$$

  for $k = 1, 2, \ldots, K$ where c_k represents the cutoff between the $(k-1)$th and kth groups

GATE Estimation as ITR Evaluation

- A natural GATE estimator

$$\hat{\tau}_k = \frac{K}{n_1}\sum_{i=1}^{n} Y_i T_i \hat{f}_k(X_i) - \frac{K}{n_0}\sum_{i=1}^{n} Y_i(1-T_i)\hat{f}_k(X_i),$$

where $\hat{f}_k(X_i) = 1\{s(X_i) \geq \hat{c}_k(s)\} - 1\{s(X_i) \geq \hat{c}_{k-1}(s)\}$

- Rewrite this as the PAPE:

$$\hat{\tau}_k = K\Bigg\{\underbrace{\frac{1}{n_1}\sum_{i=1}^{n} Y_i T_i \hat{f}_k(X_i) + \frac{1}{n_0}\sum_{i=1}^{n} Y_i(1-T_i)(1-\hat{f}_k(X_i))}_{\text{estimated PAV}} \underbrace{- \frac{1}{n_0}\sum_{i=1}^{n} Y_i(1-T_i)}_{\text{no one gets treated}}\Bigg\}$$

- We can use our previous results!

Inference for the Estimated GATE

- Exact variance for sample splitting case (bias is negligible):

$$\mathbb{V}(\hat{\tau}_k) = K^2 \left\{ \underbrace{\frac{\mathbb{E}(S_{k1}^2)}{n_1} + \frac{\mathbb{E}(S_{k0}^2)}{n_0}}_{\text{usual variance}} - \underbrace{\frac{K-1}{K^2(n-1)} \kappa_{k1}^2}_{\text{small adjustment term}} \right\},$$

  where $S_{kt}^2 = \sum_{i=1}^n (Y_{ki}(t) - \overline{Y_k(t)})^2/(n-1)$ and $\kappa_{kt} = \mathbb{E}(Y_i(1) - Y_i(0) \mid \hat{f}_k(\mathrm{X}_i) = t)$ with $Y_{ki}(t) = \hat{f}_k(\mathrm{X}_i) Y_i(t)$, and $\overline{Y_k(t)} = \sum_{i=1}^n Y_{ki}(t)/n$, for $t = 0, 1$

- Asymptotic sampling distribution:

$$\frac{\hat{\tau}_k - \tau_k}{\sqrt{\mathbb{V}(\hat{\tau}_k)}} \xrightarrow{d} N(0,1)$$

- Generalizes to cross-fitting case

Two Nonparametric Tests of Heterogeneity

1. Treatment effect heterogeneity:
   - Null hypothesis

$$H_0 : \hat{\boldsymbol{\tau}} \;=\; (\hat{\tau}_1 - \hat{\tau}, \cdots, \hat{\tau}_K - \hat{\tau})^\top$$

   - Reference distribution

$$\hat{\boldsymbol{\tau}}^\top \boldsymbol{\Sigma}^{-1} \hat{\boldsymbol{\tau}} \;\xrightarrow{d}\; \chi^2_K$$

2. Rank-consistent treatment effect heterogeneity:
   - Null hypothesis

$$H_0^* : \tau_1 \leq \tau_2 \leq \cdots \leq \tau_K$$

   - Reference distribution

$$(\hat{\boldsymbol{\tau}} - \boldsymbol{\mu}^*(\hat{\boldsymbol{\tau}}))^\top \boldsymbol{\Sigma}^{-1} (\hat{\boldsymbol{\tau}} - \boldsymbol{\mu}^*(\hat{\boldsymbol{\tau}})) \xrightarrow{d} \bar{\chi}^2_K$$

   where

$$\boldsymbol{\mu}^*(\boldsymbol{x}) \;=\; \underset{\boldsymbol{\mu}}{\operatorname{argmin}} \|\boldsymbol{\mu} - \boldsymbol{x}\|_2^2 \quad \text{subject to } \mu_1 \leq \mu_2 \leq \cdots \leq \mu_K,$$

   with $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_K)^\top$ and $\boldsymbol{x} \in \mathbb{R}^K$

A Simulation Study

- 2016 ACIC competition (Dorie *et al.*, 2019)
- Sample size $n = 4,802$ and 58 covariates, taken from a real study
- We generate data sets using their data generating process

- Sample size: $n = 100, 500$, and $2,500$
- Number of groups: $K = 5$
- Sample splitting: trained on the original ACIC data
- Cross-fitting: 5-fold
- ML algorithms: BART, Causal Forest, and Lasso
- Finite sample properties (sample splitting and cross-fitting)
    1. GATE estimation
    2. Nonparametric tests (treatment effect homogeneity $\rightsquigarrow$ false; rank-consistency $\rightsquigarrow$ true)

Sample-Splitting Case: GATE

Estimator	truth	$n_{\text{test}} = 100$ bias	s.d.	coverage	$n_{\text{test}} = 500$ bias	s.d.	coverage	$n_{\text{test}} = 2500$ bias	s.d.	coverage
Causal Forest										
$\hat{\tau}_1$	2.164	0.034	2.486	93.8%	0.041	1.071	95.0%	0.007	0.467	96.0%
$\hat{\tau}_2$	4.001	0.011	2.551	93.7	−0.060	1.183	94.4	−0.002	0.510	95.3
$\hat{\tau}_3$	4.583	−0.018	2.209	94.0	−0.003	0.956	96.4	0.020	0.421	95.8
$\hat{\tau}_4$	4.931	−0.077	2.500	94.6	0.001	1.138	94.3	0.003	0.517	95.6
$\hat{\tau}_5$	5.728	−0.058	3.332	96.0	−0.010	1.499	95.0	−0.009	0.661	95.2
BART										
$\hat{\tau}_1$	2.092	0.016	3.188	94.0%	−0.014	1.402	96.2%	0.009	0.626	95.8%
$\hat{\tau}_2$	3.913	0.127	2.918	95.1	0.028	1.388	94.0	−0.003	0.618	95.3
$\hat{\tau}_3$	4.478	−0.077	2.218	94.3	−0.041	0.968	95.0	−0.001	0.425	95.1
$\hat{\tau}_4$	5.042	−0.154	2.366	94.2	0.014	1.106	95.8	0.015	0.495	95.4
$\hat{\tau}_5$	5.881	−0.019	2.510	94.7	−0.019	1.104	94.4	−0.000	0.489	95.0
LASSO										
$\hat{\tau}_1$	3.243	0.028	2.507	94.1%	0.049	1.119	95.1%	0.003	0.769	95.1%
$\hat{\tau}_2$	3.817	−0.012	1.848	93.6	−0.013	0.834	94.5	−0.000	0.684	95.4
$\hat{\tau}_3$	4.318	−0.013	2.095	94.2	−0.002	0.930	94.5	0.010	0.516	95.0
$\hat{\tau}_4$	4.788	−0.041	2.475	94.0	−0.015	1.101	94.6	−0.001	0.480	94.6
$\hat{\tau}_5$	5.241	−0.046	3.921	94.4	0.021	1.739	95.1	0.002	0.505	95.3

Cross-Fitting Case: GATE

	$n = 100$				$n = 500$				$n = 2500$			
Estimator	truth	bias	s.d.	coverage	truth	bias	s.d.	coverage	truth	bias	s.d.	coverage
Causal Forest												
$\hat{\tau}_1$	3.976	−0.053	2.971	94.0%	2.900	−0.007	1.572	95.6%	2.210	−0.007	0.594	97.7%
$\hat{\tau}_2$	4.173	−0.061	2.584	95.9	4.112	−0.038	1.075	98.2	4.057	0.011	0.541	98.6
$\hat{\tau}_3$	4.286	−0.012	2.560	96.7	4.510	−0.054	1.058	97.7	4.545	0.019	0.465	98.1
$\hat{\tau}_4$	4.400	−0.119	2.865	97.4	4.799	0.066	1.149	97.9	4.951	−0.009	0.509	98.6
$\hat{\tau}_5$	4.569	0.140	3.447	94.1	5.086	0.001	1.620	96.0	5.643	−0.006	0.620	98.3
LASSO												
$\hat{\tau}_1$	4.191	−0.125	3.196	97.6%	4.017	−0.025	1.488	96.0%	3.752	−0.004	0.669	96.0%
$\hat{\tau}_2$	4.205	0.036	2.281	97.5	4.137	−0.069	1.027	97.9	4.028	−0.019	0.590	98.9
$\hat{\tau}_3$	4.268	−0.126	2.354	96.6	4.291	−0.019	1.000	97.9	4.323	0.037	0.488	97.5
$\hat{\tau}_4$	4.334	−0.003	2.536	96.8	4.430	0.035	1.174	96.8	4.571	0.033	0.642	97.2
$\hat{\tau}_5$	4.406	0.111	3.615	96.2	4.530	0.047	1.811	95.0	4.732	0.022	0.697	95.3

Sample-Splitting Case: Nonparametric Tests

	$n_{\text{test}} = 100$		$n_{\text{test}} = 500$		$n_{\text{test}} = 2500$	
	rejection rate	median p-value	rejection rate	median p-value	rejection rate	median p-value
Causal Forest						
H_0: *Treatment effect homogeneity*						
$n_{\text{train}} = 100$	5.2%	0.504	7.4%	0.529	19.6%	0.361
$n_{\text{train}} = 400$	9.0	0.459	22.0	0.254	74.4	0.002
$n_{\text{train}} = 2000$	13.0	0.367	40.4	0.092	96.0	0.000
H_0^*: *Rank consistency of GATEs*						
$n_{\text{train}} = 100$	4.0%	0.583	2.2%	0.624	2.2%	0.704
$n_{\text{train}} = 400$	2.8	0.687	0.2	0.820	0.2	0.907
$n_{\text{train}} = 2000$	1.2	0.707	0.2	0.852	0.0	0.967
LASSO						
H_0: *Treatment effect homogeneity*						
$n_{\text{train}} = 100$	5.8%	0.496	5.2%	0.544	9.6%	0.516
$n_{\text{train}} = 400$	7.0	0.557	4.0	0.578	10.4	0.468
$n_{\text{train}} = 2000$	6.2	0.489	9.4	0.519	26.2	0.249
H_0^*: *Rank consistency of GATEs*						
$n_{\text{train}} = 100$	4.6%	0.525	3.0%	0.584	5.4%	0.596
$n_{\text{train}} = 400$	6.0	0.494	1.8	0.600	2.4	0.687
$n_{\text{train}} = 2000$	3.2	0.608	1.4	0.698	1.2	0.851

Cross-Fitting Case: Nonparametric Tests

	$n = 100$		$n = 500$		$n = 2500$	
	rejection rate	median p-value	rejection rate	median p-value	rejection rate	median p-value
Causal Forest						
Homogeneous Treatment Effects	1.4%	0.790	4.6%	0.712	51.4%	0.041
Consistent Treatment Effects	1.4%	0.702	0.8%	0.845	0.0%	0.976
LASSO						
Homogeneous Treatment Effects	0.6%	0.880	1.8%	0.850	9.0%	0.664
Consistent Treatment Effects	1.0%	0.722	0.6%	0.769	0.2%	0.889

Empirical Application

- National Supported Work Demonstration Program (LaLonde 1986)
- Temporary employment program to help disadvantaged workers by giving them a guaranteed job for 9 to 18 months

- Data
  - sample size: $n_1 = 297$ and $n_0 = 425$
  - outcome: annualized earnings in 1978 (36 months after the program)
  - 7 pre-treatment covariates: demographics and prior earnings

- Setup
  - ML algorithms: Causal Forest, BART, and LASSO
  - Sample-splitting: 2/3 of the data as training data
  - Cross-fitting: 3 folds
  - 5 fold cross-validation for tuning parameters

GATE Estimates (in 1,000 US Dollars)

	$\hat{\tau}_1$	$\hat{\tau}_2$	$\hat{\tau}_3$	$\hat{\tau}_4$	$\hat{\tau}_5$
Sample-splitting					
Causal Forest	3.40	0.13	−0.85	−1.91	7.21
	[−1.29, 3.40]	[−5.37, 5.63]	[−5.22, 3.52]	[−5.16, 1.34]	[1.22, 13.19]
BART	2.90	−0.73	−0.02	3.25	2.57
	[−2.25, 8.06]	[−5.05, 3.58]	[−3.47, 3.43]	[−1.53, 8.03]	[−3.82, 8.97]
LASSO	1.86	2.62	−2.07	1.39	4.17
	[−3.59, 7.30]	[−1.69, 6.93]	[−5.39, 1.26]	[−2.95, 5.73]	[−2.30, 10.65]
Cross-fitting					
Causal Forest	−3.72	1.05	5.32	−2.64	4.55
	[−6.52, −0.93]	[−2.28, 4.37]	[2.63, 8.01]	[−5.07, −0.22]	[1.14, 7.96]
BART	0.40	−0.15	−0.40	2.52	2.19
	[−3.79, 4.59]	[−2.54, 2.23]	[−3.37, 2.56]	[−0.99, 6.03]	[−0.73, 5.11]
LASSO	0.65	0.45	−2.88	1.32	5.02
	[−3.65, 4.94]	[−3.28, 4.18]	[−5.38, −0.38]	[−1.83, 4.48]	[−0.14, 10.18]

Nonparametric Tests

	Causal Forest		BART		LASSO	
	stat	p-value	stat	p-value	stat	p-value
Sample-splitting						
Homogeneous Treatment Effects	9.78	0.082	2.76	0.737	5.26	0.362
Rank-consistent Treatment Effects	3.07	0.323	1.13	0.657	3.14	0.302
Cross-fitting						
Homogeneous Treatment Effects	30.29	0.000	2.32	0.803	10.79	0.056
Rank-consistent Treatment Effects	0.06	0.691	0.04	0.885	0.45	0.711

Concluding Remarks

- Causal machine learning is everywhere
  - estimation of heterogeneous treatment effects (HTEs)
  - development of individualized treatment rules (ITRs)
- Inference about HTEs and ITRs has been largely model-based
  - We show how to experimentally evaluate HTEs and ITRs
  - No modeling assumption or asymptotic approximation is required
  - Complex machine learning algorithms can be used
  - Applicable to cross-fitting estimators
  - Simulations: good small sample performance
- Ongoing extension: dynamic ITRs
- Open source software: evalITR: Evaluating Individualized Treatment Rules at CRAN https://CRAN.R-project.org/package=evalITR